OPTIMAL MODAL CONTROL OF POWER SYSTEMS

A Thesis Submitted
In partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

by

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to the

DEPARTMENT OF ELECTRICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY KANPUR

JULY, 1976

Dedicated to
my beloved parents and wife.

CERTIFICATE

It is certified that this work entitled 'Optimal Modal Control of Power Systems' by D.N. Nigam has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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ACKNOWLEDGEMENT

I take this opportunity to express my deep sense of gratitude to my supervisors, Dr. M.A. Pai and Dr. S.S. Prabhu who had initiated me into the problem and provided the much needed infallible guidance. Their sincere advice and the keen interest during the course of investigations have been a regular source of constant encouragement to me.

Also my thanks are due to all my fellow-research scholars especially M/S. C.L. Narayana, S.R. Paranjothi, S.D. Varwandkar, P.J. Rao and Shudhakar Shetty for some useful discussions and the help rendered in the preparation of this thesis.

I would also like to thank Mr. S.V. Sirbhai for the care and interest taken in typing and Mr. Triveni Tiwari for skilful cyclostyling.

Lastly, I should appreciate my wife Shashi and children for their forbearance throughout the venture of this work.

DAYA NARAIN NIGAM

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LIST OF PRINCIPAL SYMBOLS

Modal Control Theory

A : nxn matrix of the uncontrolled system

B : nxm matrix of the control

$\underline{x}$: n-vector of state variables

$\underline{u}$: m-vector of control variables

λ : eigenvalue of the matrix A

Λ : diagonal matrix of eigenvalues

W : eigenvector matrix of A

V : eigenvector matrix of A^T

v_i : eigenvector of A^T associated with i^{th} eigenvalue λ_i

k : controller feedback loop gain

F : mxn modal controller feedback matrix

M : ℓxm mode controllability matrix

ℓ : number of eigenvalues to be assigned

A superscript T denotes Transpose.

A bar below a symbol signifies a vector.

A superscript $*$ denotes conjugation.

Power System

x_d : synchronous reactance d-axis

x'_d : transient reactance d-axis

x_q : synchronous reactance q-axis

x_{ad} : mutual reactance on d axis

x_{aq} : mutual reactance on q axis

e_t : measured terminal voltage

E_{ref} : reference signal to exciter

v_f : field voltage

e_{fd} : field voltage of d-axis field winding

δ : Torque angle in radians

μ_s : voltage control feedback loop gain

μ_a : governor actuator gain

μ_e : exciter gain

τ_s : voltage control feedback loop time constant

τ_a : governor time constant

τ_e : exciter time constant

τ_g : gate time constant

τ'_{do} : open circuit time constant of the field

P_i : mechanical power input

P_e : power of electro-mechanical energy conversion

ω_o : synchronous speed (377 rad./sec.)

ω : speed (rad./sec.)

β : time scaling factor

T_{ing} : change in gate (valve) position

T_D : Damping constant

T_M : Mechanical Torque

L.F.C. Problem

P_d : Load demand

P_{tie} : tie line power

P_c : speed changer position

P_g : generation

X_{gv} : governor valve position

T_{12}^{*} : tie line constant

H : inertia constant

D : load frequency constant

f : frequency

T_t : turbine time constant

T_{gv} : speed governor time constant

A dot over a symbol denotes differentiation with respect to time.

p(.) denotes the differentiation of (.) with respect to time.

Δ(.) denotes the increment in (.).

o denotes the nominal value of a variable (subscript).

SYNOPSIS

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OPTIMAL MODAL CONTROL OF POWER SYSTEMS

Recent decades have witnessed a phenomenal growth in the size and interconnection of power systems all over the world. The problems of design and operation of these systems are complex and require sophisticated techniques, especially from the area of system theory, for their solution.

Production of large amounts of power at low cost has necessitated employment of generators of high capacity, exploitation of remote hydro-electric sources and transmission of bulk power by e.h.v. transmission lines, and increased interconnections to obtain economy and reliability. The fundamental problems in the design and operation of such systems are those associated with system stability, economic operation, reliability of supply to the consumers, security and operation within the constraints such as those on system voltages and frequency. Many of these problems can be posed as problems in the area of automatic control. Consequently there has been considerable activity, in

recent years, in formulating these problems and determining efficient solution techniques to them using techniques of modern control theory.

Earlier approaches to improve the power system stability by the employment of fast acting exciter voltage regulating systems resulted in improved transient stability of the system. However, these systems generally exhibited poor damping of machine angle swings. These swings cause variation in system voltages and frequency and in extreme cases may even result in system instability. An appropriately designed feedback controller may overcome this problem. Therefore, in the recent past, researchers have paid great attention towards the development of suitable controls to damp out quickly the machine angle oscillations and at the same time retain the improvement in the transient stability. A major difficulty with this problem is that the system equations are nonlinear and of high order. An approach which is usually employed makes use of a linear system model valid for small departures of the system from its nominal operating conditions. Optimal infinite time linear regulator theory is then used to obtain linear, time-invariant feedback laws. The following difficulties are encountered in this approach. (a) It is difficult to determine apriori what the state and control weighting matrices, Q and R respectively, should be to obtain

satisfactory closed loop system performance. It is necessary to solve the optimal control problems successively, for various Q and R matrices, till satisfactory system performance is obtained. The method does not provide the designer with a 'feel' for the problem, and is computationally unattractive. (b) The method requires accessibility of all states of the system for measurement. The use of observers which overcomes this drawback, introduces its own computational complexity. (c) The system should be completely state controllable. (d) The feedback law obtained for one operating point of the system may be inappropriate at others. (e) Furthermore, there is a problem associated with feedback gains. From the practical point of view, large feedback gains should be avoided. The approach referred to above does not provide direct control over the feedback gains.

This thesis attempts to overcome some of the problems mentioned above by developing suitable techniques stemming from modal control theory. Like the classical root locus technique, modal control approach provides the designer with a 'feel' for the problem, and his engineering judgement can be better utilized for developing suitable designs. In many practical situations, system behaviour is governed mainly by a small number of 'dominant' eigenvalues. A suitable design can usually be obtained by determining a feedback controller

which shifts these eigenvalues far enough into the left half plane without disturbing the locations of the non-dominant eigenvalues. Complete state controllability is not necessary for achieving this objective. This thesis reports a technique for minimizing feedback gains to obtain optimal modal controllers and procedures which facilitate determination of modal controllers, optimal and non-optimal, for high order systems. In the case of high order systems, grouping of eigenvalues and use of appropriate control inputs to shift these eigenvalues is proposed. Using these techniques and coupled with the knowledge regarding sensitivity of the eigenvalues to system parameter perturbations, wide range modal controllers have been developed. These controllers give satisfactory system performance for a range of operating conditions. The thesis also considers design of incomplete state feedback and output feedback modal controllers. Modal control technique developed has been also applied to design an output feedback dynamic controller.

The design techniques developed pertain to finite-dimensional linear, time-invariant systems; they can therefore be applied in situations where the physical systems can be adequately represented by such mathematical models. In addition to applying the technique to the control of a power system with its associated controls, the

thesis also illustrates the use of some of the techniques to the design of Load Frequency Controllers and controllers for high order systems. Specifically modal controller design of a forty-first order system describing a chemical plant has been given for the purpose of illustration.

A chapterwise summary of the work reported in this thesis is given below:

The problem of design of feedback controllers for power system is introduced in Chapter 1. A review of the recent developments in this area is given.

Chapter 2 is devoted to the development of the state space model for a power system which is used for illustrating the techniques of control developed in Chapters 3, 4, 5 and 6. The power system considered consists of a synchronous generator connected to an infinite bus through a transmission line and has an exciter-voltage regulator and a turbine speed governor. The generator is driven either by a hydraulic turbine, or a steam turbine (as a prime-mover). The nonlinear equations describing the system dynamics are derived. Linear models are obtained by linearizing the nonlinear equations. The model for the design of Load Frequency Control scheme for a two area system is also briefly described. The control problems associated with the above models have been enunciated.

Chapter 3 introduces the concept of optimal modal control by suggesting appropriate performance indices. The indices involve minimization of feedback gains. The closed loop eigenvalues are decided as in classical control theory based on 'feel' of the problem. A solution technique for designing an optimal modal controller is then described. Using this algorithm optimal modal controllers have been designed for a power system. The algorithm has also been applied to the design of a load frequency controller for a two area problem. The responses are compared with conventional schemes as well as uncontrolled responses.

The problem of designing controllers for high order systems are discussed in Chapter 4. A straightforward application of the technique developed in Chapter 3 will result in high feedback gains. This will be so in spite of the feedback gain optimization, due to the constraints imposed on the structure of the feedback matrix. By relaxing the constraint, an alternative design procedure for optimal modal control of large systems has been developed. This procedure is based on grouping of eigenvalues and moving these groups to desired locations successively through appropriately chosen inputs. Modification of this procedure which results in advantages in computation as well as implementation has been indicated. Three illustrative problems for the design of controllers for power systems have been solved and the results discussed.

To demonstrate the efficacy of the algorithms developed, they have been applied to the controller design for a chemical plant having fortyone state variables and eight control inputs.

Linear regulators for power systems are usually designed for a particular set of system operating conditions. However, these conditions change with the load demand on the system. In Chapter 5, the design of a wide-range modal controller is developed using modal control theory and eigenvalue sensitivity analysis. The design reduces the variation in closed loop pole locations with change in operating conditions. A numerical example of the design of the wide range controller has been given and the practical implementation discussed.

Modal controller design requires feedback of all the state variables. In practice it may be difficult or impossible to do so. A technique to design modal controller with accessible state or output feedback is described in Chapter 6. The chapter also includes the design, using modal control approach of a dynamic output feedback controller.

Chapter 7 reviews the work reported and contribution made in this thesis and concludes with an assessment of the scope for further work in this area.

CHAPTER 1

INTRODUCTION

1.1 POWER SYSTEM CONTROL PROBLEM

The phenomenal growth in the size of power systems and the increase in interconnections to improve reliability of bulk power supply results in system stability (both transient and dynamic) problems. By transient stability we imply system response to large disturbances and dynamic stability pertains to small signal behaviour around an operating point. Both phenomena are of concern to system planning and operating people. Construction of super thermal stations, commissioning of large capacity generating units, use of extra-high-voltages for large distance transmission, hydraulic generation at remote locations and the design of modern alternators with large synchronous reactances reduce the stability margin. To improve the transient stability of the power system, various control strategies have been investigated [1-8]. Considerable improvement in the transient stability has been obtained by using fast acting excitation systems [2]. Though the damping of rotor angle oscillations is initially more, it becomes poor after the first cycle. It is also known that modern fast acting AVR's and governors while contributing to improved steady state response and dynamic control can

aggravate the problem of small signal dynamic stability. The current practice of running the generators at high power-factor and the generation of reactive power due to adoption of extra-high voltage transmission lines and cables, requires that more emphasis be placed on the development of compensating controls to stabilize the system.

Power system simulation studies are used to extend the simulation of power system behaviour for several minutes after the first swing and provides the transition between the first swing transient stability which gives immediate post-fault behaviour and the load flow which describes the steady state of the post-fault conditions. A properly planned and operated power system permits satisfactory transient performance following a disturbance and then returns to a desirable steady state. Stability studies are done to see that the system meets the requirements [9,10].

After a disturbance occurs on the system, the state of the system such as voltage levels, frequency, tie line power flows and line currents will change. In extreme cases, the synchronous machines in the system may become unstable. Control of a power system to bring back the system state to normal operating condition is an essential requirement [11]. Moreover, the control actions must be such as to restore the state to the normal condition in

the shortest possible time after a disturbance occurs on the system.

In the methods of power system control the deviation in the states such as voltage, frequency and machine angles are detected by means of sensors. Depending on the error, the voltage regulator and excitation system and the governor and turbine system take corrective steps to keep the system states within limits. In recent years, the effect of excitation and speed governing systems on power system performance has been studied extensively. Improvements in the system performance through subsidiary feedback applied to the basic voltage or frequency regulating system have been investigated. The feedback of supplementary signals obtained from rotor angle, rotor speed and terminal voltage into the excitation system has been shown to be very effective [3,12,13]. Classical control tools such as Routh-Hurwitz, Nyquist and Root locus technique have also been used to analyse the dynamic stability phenomena of a power system [14-16]. However, these techniques are cumbersome to apply even to a simple power system and almost impossible for large multimachine interconnected power systems.

1.2 MODERN TRENDS AND THE STATE OF ART

The availability of digital computers has made it possible to undertake stability studies of large interconnected power systems. Considerable interest

has been shown in recent years [6-8,17-47] in the application of the results of modern control theory to the optimization of power system performance. The application of the modern control theory has enabled the optimization to be carried out in a systematic and logical manner. The philosophy of system optimization requires that the performance index (J) in the form of the integral of a function of the system variables, $\underline{x}$, and the control functions, $\underline{u}$, be defined and minimized over a specific time interval. The state variable transients and control actions are minimized simultaneously.

The dynamics of the power system are represented by a set of nonlinear differential equations

$$\dot{\underline{x}} = \underline{f}(\underline{x}, \underline{u}, t) \qquad (1.1)$$

The optimal control law is obtained by solving a set of differential equations obtained by applying Pontryagin's minimum principle. Some researchers [17-19,48] attempted to solve the nonlinear power system optimization directly. In references [17] and [18] the optimal control has been obtained as a function of time, using a Newton-Raphson gradient minimization and quasilinearization technique respectively. Rajagopalan and Hariharan [48] proposed a bang-bang form of excitation input and the switching time is calculated to minimize a performance index by a

hill climbing method. The open loop form of control calculated by the above procedures is highly sensitive to variation of system operating conditions. Quasi-Newton method of function minimization to derive closed loop constant gain feedback control to optimize nonlinear system response has been attempted in reference [19]. In this approach, the nonlinear equations (1.1) are to be solved to evaluate the performance index.

The solution of nonlinear eqns. (1.1) is computationally difficult. However, for studying the transients due to small disturbances, the original nonlinear system can be linearized about the operating point to the linear matrix form

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{1.2}$$

where the variables now denote their deviation from their nominal operating conditions.

A linear regulator with constant feedback gains, can be designed, using modern control theory, to obtain satisfactory closed loop system performance. There are mainly two approaches to the design of linear regulators: (1) procedures based on linear optimal regulator theory and (2) procedures based on pole assignment techniques.

1.2.1 Optimal Control of Synchronous Machine:

Most researchers [6-8,20-36] have obtained the optimal control for a power system through the application

of linear regulator theory which involves the solution of the matrix Riccati equation. The optimal control law so obtained requires the feedback of all the state variables. The direct measurement of all the state variables may not be possible in a system. Even though an observer may be constructed to estimate the states, it may not be possible to implement it. In a realistic power system, to provide feedback of certain measurable states, may not be practicable. For example, in a hydro system the equipment to measure the change in water level is situated far away from the generating station. Transmission of such signals may be expensive and noise is introduced in transmitting such signals. These constraints prompted the design of controllers based on the measurement of available states or outputs [21-25]. Davison and Rau [21] obtained feedback laws in terms of the various combinations of outputs using parameter optimization method and the solution of Liapunov's matrix equation. DeSarkar et al [22] and Pai et al [39] designed output feedback controllers by detecting the feedback from undesirable states. However, controller designs based upon above approach may not give satisfactory response and in extreme cases the system may even become unstable. In reference [23] a suboptimal control policy obtained through a reduced model which contains only dominant eigenvalues of the original system

is proposed. Habibullah et al [24] obtained the output feedback controller by choosing a state space model having outputs as state variables. Raina et al [25] designed a feedback control system based on quantities easily measurable at the synchronous generator locations. Furthermore, the designs of references [24,25] have been shown to give satisfactory response for a wide range of operating conditions.

The controllers designed by linearizing the system equations about a particular operating point may not give satisfactory response for other operating conditions. An additional correcting feedback signal to compensate for small variation in operating conditions has been suggested [26]. Optimal regulator theory incorporating the trajectory sensitivity function in the performance index has been suggested by Elmetwally and Rao [27,30,32] to design low sensitivity excitation control. Design based on linear optimal regulator theory and eigenvalue search technique has been also suggested [24], to design wide power range controllers for power systems.

1.2.2 Load Frequency Control Problem:

Ever since the pioneering work of Elgerd and Fosha [37,38], there has been continuing interest in the multiarea load frequency control problem [39-45]. Venkat et al [41] designed load frequency controller

for systems with unknown disturbances utilizing the state augmentation approach of Smith and Davison. Ramamoorty et al [43] and Rao et al [44] designed regulators for two-area power systems via Liapunov's second method and utilizing minimum settling time theory. The importance of the dominant time constant of the closed loop system in designing the regulators has been emphasized by them. Furthermore, Rao et al [44] suggested a criterion for choosing the frequency bias setting. In a recent paper Elmetwally and Rao [45] designed a controller with assumed constrained structure to study the decentralized control of multiarea load frequency control problem. Their design of local controller is easily implementable and gives comparable results with those obtained by centralized controller.

1.2.3 Pole Assignment Procedures:

Recently in the literature, emphasis has been given to the locations of the closed loop system poles, while designing the regulators for power system control [24,31,36,49-51]. Elangovan et al [36] suggested a method to design controllers with prescribed eigenvalues. In their approach, the state weighting matrix in the performance index is selected to give desired closed loop eigenvalues locations. Using pole assignment procedure, Seraji [31] designed controller for a single input synchronous generator. Pai et al [49] applied modal control theory

for designing controllers for a power system with two inputs. However, their technique utilizes only one of the two available inputs to shift one or more eigenvalues at a time.

1.3 OBJECTIVE AND SCOPE OF THE THESIS

1. In the optimal linear regulator theory, it is necessary to systematically change the state and control weighting matrices, Q and R respectively, in the performance index (J), till a satisfactory closed loop system behaviour is obtained. It is therefore better to employ design techniques which directly yield the required closed loop system eigenvalue locations. This would avoid the problem of having to iteratively solve a sequence of optimal linear regulator problems. Furthermore, the optimal linear regulator theory does not provide the designer with a 'feel' for the problem, and is computationally unattractive, especially for high order systems. Pole-placement techniques coupled with a performance criteria for optimization is a desirable goal to achieve.

2. Controller designed for power systems using the optimal infinite time linear regulator theory or modal control techniques [52-55] requires feedback of all the states of the system. It is seldom that all of them can be monitored directly in the case of power systems. The use of observers though overcomes this problem, introduces

its own computational complexity. Furthermore, the requirements that the system should be completely controllable for designing optimal controller and completely observable for the design of observer are restrictive. In certain systems the output variables which are linear combination of state variables may only be available for measurements. Hence controllers based on output feedback are practically desirable.

3. In the case of large scale power systems, there is a problem associated with the feedback gains. The complexity of the problem increases in arranging feedback to all the control inputs. Excessive computational efforts may be required in optimizing the controller design for large systems with more number of control inputs. A technique in moving eigenvalues in groups using one or more selected inputs which at the same time minimizes feedback gains will result in an improved controller.

4. The linear regulators (optimal and modal controllers) are designed for a particular operating conditions. In a power system, the operating conditions do change. Therefore, the controllers designed for one operating condition may be inappropriate for other operating points. The need for a wide range controller is therefore clear.

The thesis addresses itself to the problems enumerated above. Accordingly the objectives of the thesis are set forth as follows:

(i) to develop suitable techniques stemming from modal control theory for the design of modal controllers for power systems so as to minimize the feedback gains. The techniques should be such that the designer's engineering judgement can be better utilized for developing suitable designs.

(ii) to suggest a suitable procedure using eigenvalue grouping technique for minimizing feedback gains to obtain optimal modal controllers. Computationally simple procedures for the design of modal controllers, optimal and non-optimal for high order systems are to be developed. The possibilities of designing controllers with feedback to small number of inputs without appreciably affecting the feedback gains are explored.

(iii) to propose control laws using sensitivity techniques so that, the effect of the wide range change in operating conditions is small on the locations of the closed loop pole locations.

(iv) to present design techniques for designing physically realizable modal controllers with easily accessible states or output feedback.

The chapterwise summary of the various chapters in this thesis are given in the synopsis.

CHAPTER 2

MODELLING OF POWER SYSTEMS

2.1 INTRODUCTION

An adequate mathematical model is a prerequisite for any application of modern control theory to design controllers for power systems. With the availability of fast and large digital computers, sufficiently complex power system models can be analysed. In general power system model is nonlinear, comprising of the differential equations of the synchronous machine and its controls and the algebraic equations governing the transmission network. Application of optimal control theory to such models is difficult. Therefore the usual practice is to linearize the model around an operating point and then develop the control laws. Linear models are in wide use due to the ease with which one can apply the results of multivariable control theory. High order models yield better accuracy but difficulties arise in computation and practical implementation of control strategies. It is therefore necessary to strike a balance between ease of implementation and engineering accuracy which is desirable. The literature in the area of dynamic modelling of power systems is quite profuse [56-65]. Hence it is not proposed to discuss the modelling aspect in detail.

The basic power system components to be modelled are (i) transmission line, (ii) synchronous machine, (iii) voltage regulator and excitation system and (iv) primemover and governor system. In dynamical studies the variation in the rotor angles is essentially a low frequency phenomenon. Hence the transients in the transmission network are neglected and steady state phasor representation of the transmission network is used. The synchronous machine is represented by Park's equations [56]. To reduce the complexity of the model, two simplifications are made (i) the stator transients are neglected and (ii) the speed voltage terms are evaluated with ω equal to ω_o (synchronous speed). The first assumption is consistent with the representation of the transmission network. It is further assumed that the resistance of the transmission line and the effect of magnetic saturation in the machine are neglected. The voltage regulator and the exciter system is represented by a single time constant. The primemover and governor system is represented by two time constants [63], which is simplified to a single time constant in the case of fast acting governor and pilot valve system.

Various aspects of modal control theory developed in this thesis have been applied to basically three dynamical models of the power system. (i) Power system considered by Yu et al [6], (ii) Power system considered

by Davison and Rau [21] and (iii) Two area load frequency control system considered by Fosha and Elgerd [38].

A brief description, the performance equations, the linearized model and the data for the above systems are given in the subsequent sections. The details are contained in the appropriate references.

2.2 POWER SYSTEM MODELS

2.2.1 Power System Considered by Yu et al [6]:

The power system considered consists of a synchronous generator driven by a hydraulic turbine, having an exciter-voltage regulator and a turbine speed governor. The generator is connected to an infinite bus via a long transmission line [Figure 2.1].

The increment in field voltage i.e. Δv_f is controlled through a signal u_1. This signal is applied through the voltage control feedback loop having transfer function $[\mu_s/(1 + \tau_s p)]$, to the summing junction of the exciter-voltage regulator system [Figure 2.2].

Increment in the power input ΔP_i to the machine is controlled by a control signal u_2. This control signal is applied via a servomotor represented by a simple transfer function $[\mu_a/(1 + \tau_a p)]$ to the summing junction of the governor system [Figure 2.3].

The synchronous machine has a three phase balanced winding and the resistance of the winding is smaller than

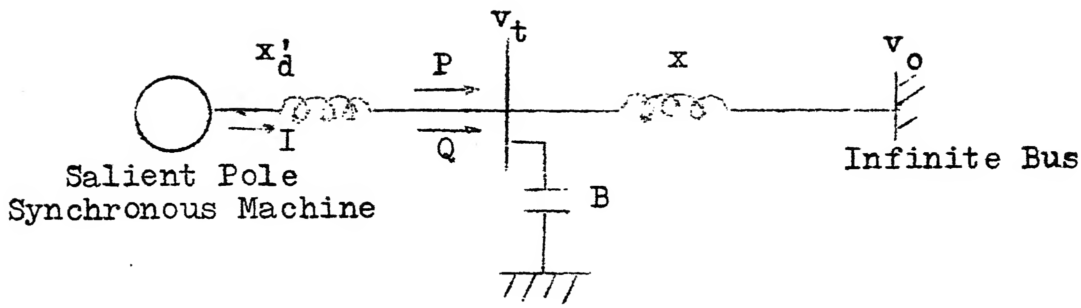

FIGURE 2.1 SYNCHRONOUS MACHINE INFINITE BUS SYSTEM

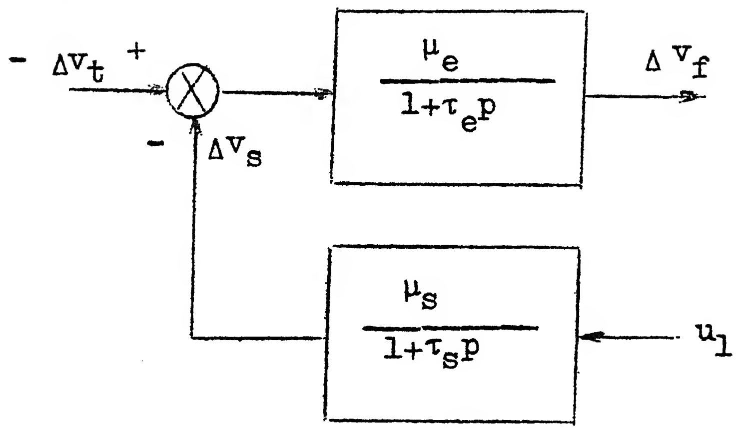

FIGURE 2.2 EXCITER-VOLTAGE REGULATOR SYSTEM

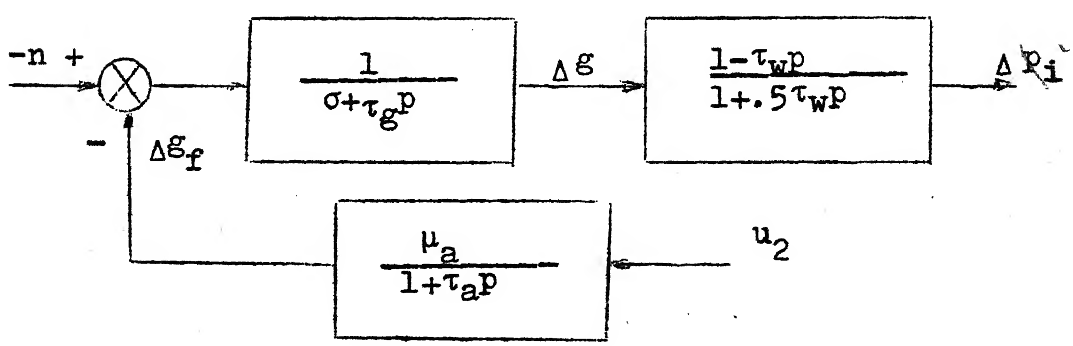

FIGURE 2.3 GOVERNOR-HYDRAULIC SYSTEM

its reactance. The rotor carries the field winding only. The machine terminal conditions (Figure 2.1) give in d-q components the relation, [6]

$$(v_d + jv_q)(jB) + [(v_d + jv_q) - (v_o \,\mathrm{Sin}\delta + jv_o \mathrm{Cos}\delta)]/jx$$
$$= i_d + ji_q \tag{2.1}$$

The park's equations to be considered are

$$v_d = -\,\omega\Psi_q \tag{2.2}$$

$$v_q = \omega\Psi_d \tag{2.3}$$

$$\omega\Psi_q = -x_q\, i_q \tag{2.4}$$

$$\omega\Psi_d = \frac{v_f}{1+\tau'_{do}p} - \frac{1+\tau'_d p}{1+\tau'_{do}p}\, x_d\, i_d \tag{2.5}$$

where

v_o = infinite bus voltage

v_f = a field voltage

v_d, v_q = d and q component of v_t, respectively

i_d, i_q = d and q component of armature current i, respectively,

Ψ_d, Ψ_q = d-axis and q-axis flux linkages, respectively

δ = torque angle (radians)

ω_o = synchronous speed (rad/sec.)

ω = speed (rad/sec.)

$x_q, x_d, \tau'_{do}, \tau'_d, x'_d$ = appropriate synchronous machine constants.

Let

$$\tau = \beta\ t \tag{2.6}$$

where β is the time scaling factor. Then

$$n \triangleq \frac{\Delta\omega}{\beta} \tag{2.7}$$

$$\frac{d\ \Delta\delta}{d\tau} = n \tag{2.8}$$

$$\frac{dn}{d\tau} = \frac{1}{\beta^2 M}[\Delta P_i - \beta\ D\ n - \Delta P_e] \tag{2.9}$$

where

ΔP_i = increment in the mechanical power input

ΔP_e = increment in the power of the electro-mechanical energy conversion

D = damping coefficient.

The increment in the mechanical power input is approximated by [6],

$$\Delta P_i = \Delta g + 1.5 \Delta h \tag{2.10}$$

where Δg and Δh are the increments in the gate opening and the water head respectively. The power of electro-mechanical energy conversion is given by the expression

$$P_e = \frac{v_o \Psi_f \text{ Sin } \delta}{X'_d\ \tau'_{do}} + \frac{(x'_d - x_q)}{2X'_d\ X_q} v_o^2 \text{ Sin } 2\delta \tag{2.11}$$

where $P_e = P_o + \Delta P_e$

P_o = the power delivered by the machine at the operating conditions.

Linearizing eqn.(2.11) around the operating point, one gets

$$\Delta P_e = \left[\frac{v_o \, \Psi_{fo} \, \cos \delta_o}{X'_d \, \tau'_{do}} + \frac{(x'_d - x_q)}{X'_d \, X_q} v_o^2 \cos 2\delta_o \right] \Delta \delta + \frac{v_o \, \sin \delta_o}{X'_d \, \tau_{do}} \Delta \Psi_f \qquad (2.12)$$

Substituting eqns. (2.10) and (2.12) in eqn.(2.9), one gets

$$\frac{dn}{d\tau} = \frac{1}{\beta^2 M} \left\{ \left[- \frac{v_o \, \Psi_{fo} \, \cos \delta_o}{X'_d \, \tau'_{do}} - \frac{(x'_d - x_q)}{X'_d \, X_q} v_o^2 \cos 2\delta_o \right] \Delta \delta - \beta D n - \frac{v_o \, \sin \delta_o}{X'_d \, \tau'_{do}} \Delta \Psi_f + \Delta g + 1.5 \Delta h \right\} \qquad (2.13)$$

The rate of change of flux linkages of the field winding is given by

$$\frac{d \Psi_f}{d\tau} = \frac{v_f}{\beta} - \frac{X_d \Psi_f}{\beta X'_d \tau'_{do}} + \frac{(x_d - x'_d)}{\beta \, X'_d} v_o \cos \delta \qquad (2.14)$$

On linearization around the operating point, the above equation yields

$$\frac{d \, \Delta\Psi_f}{d\tau} = \frac{\Delta v_f}{\beta} - \frac{X_d \, \Delta\Psi_f}{\beta \, X'_d \, \tau'_{do}} - \frac{(x_d - x'_d)}{\beta \, X'_d} v_o \sin \delta_o \, \Delta \delta \qquad (2.15)$$

where

$x'_d \triangleq$ direct axis transient reactance of the synchronous machine

$X'_d \triangleq (1 - x B) \, x'_d + x$

$X_q \triangleq (1 - x B) \, x_q + x$

$$X_d \overset{\Delta}{=} (1 - x B) x_d + x$$

B = transmission line susceptance (Figure 2.1).

The exciter voltage regulator system equations in state variable forms are obtained from Figure 2.2 as

$$\frac{d\Delta v_f}{d\tau} = - \frac{\Delta v_f}{\beta\tau_e} + \frac{\mu_e}{\beta\tau_e}(-\Delta v_t - \Delta v_s) \tag{2.16}$$

$$\frac{d\Delta v_s}{d\tau} = - \frac{\Delta v_s}{\beta\tau_s} + \frac{\mu_s}{\beta\tau_s} u_1 \tag{2.17}$$

where

$$v_t^2 = v_d^2 + v_q^2$$

and the first order variation in v_t is approximated as

$$\Delta v_t = \frac{v_{do}}{v_{to}} \Delta v_d + \frac{v_{qo}}{v_{to}} \Delta v_q \tag{2.18}$$

On substituting for Δv_d and Δv_q, the above equation reduces to

$$\Delta v_t = \frac{v_o}{v_{to}}\left[\frac{x_q \; v_{do} \; \text{Cos}\; \delta_o}{X_q} - \frac{x_d' \; v_{qo} \; \text{Sin}\; \delta_o}{X_d'} \right] \Delta\delta + \frac{x \; v_{qo}}{v_{to} \; X_d' \tau_{do}'} \Delta\Psi_f \tag{2.18a}$$

Substituting (2.18a) in (2.16), the eqn.(2.16) reduces to

$$\frac{d\Delta v_f}{d\tau} = - \frac{\Delta v_f}{\beta\tau_e} + \frac{\mu_e}{\beta\tau_e}\left\{ - \frac{v_o}{v_{to}} \left[\frac{x_q \; v_{do} \; \text{Cos}\; \delta_o}{X_q} - \frac{x_d' \; v_{qo} \; \text{Sin}\delta_o}{X_d'}\right]\Delta\delta - \frac{x \; v_{qo}}{v_{to} \; X_d' \; \tau_{do}'} \Delta\Psi_f - \Delta v_s \right\} \tag{2.19}$$

The primemover governor system equation in state variable form are obtained from [Figure 2.3]

$$\frac{d\,\Delta g}{d\tau} = -\frac{\sigma}{\beta\tau_g}\Delta g + \frac{1}{\beta\tau_g}\left(-\frac{\beta\,n}{\omega_o} - \Delta g_f\right) \tag{2.20}$$

$$\frac{d\,\Delta g_f}{d\tau} = -\frac{\Delta g_f}{\beta\tau_a} + \frac{\mu_a}{\beta\tau_a}u_2 \tag{2.21}$$

$$\frac{d\,\Delta h}{d\tau} = -\frac{2d\,\Delta g}{d\tau} - \frac{2}{\beta\tau_w}\Delta h \tag{2.22}$$

The state variables selected for the linearized model are

$$\underline{x} = [\Delta\delta,\ n, \Delta\Psi_f,\ \Delta v_f,\ \Delta v_s,\ \Delta g,\ \Delta g_f,\ \Delta h]^T \tag{2.23}$$

and the linearized power system model is given by the eqns. (2.8), (2.13), (2.15), (2.19), (2.17) and (2.20) to (2.22) which are put in standard state space form as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{2.24}$$

where

$$\underline{u} = [u_v,\ u_g]^T \tag{2.25}$$

$$u_v = \frac{\mu_s\,u_1}{\beta\,\tau_s} \tag{2.26}$$

$$u_g = \frac{\mu_a\,u_2}{\beta\,\tau_a} \tag{2.27}$$

μ_s = voltage control feedback loop gain

μ_a = governor actuator gain

τ_s = voltage control feedback loop time constant

τ_a = governor actuator time constant

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{22} & \frac{-D}{\beta M} & a_{23} & 0 & 0 & \frac{1}{\beta^2 M} & 0 & \frac{1.5}{\beta^2 M} \\ a_{31} & 0 & a_{33} & \frac{1}{\beta} & 0 & 0 & 0 & 0 \\ a_{41} & 0 & a_{43} & \frac{-1}{\beta\tau_e} & \frac{-\mu_e}{\beta\tau_e} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1}{\beta\tau_s} & 0 & 0 & 0 \\ 0 & \frac{-1}{\omega_o\tau_g} & 0 & 0 & 0 & \frac{-\sigma}{\beta\tau_g} & \frac{-1}{\beta\tau_g} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{\beta\tau_a} & 0 \\ 0 & \frac{2}{\omega_o\tau_g} & 0 & 0 & 0 & \frac{2\sigma}{\beta\tau_g} & \frac{2}{\beta\tau_g} & \frac{-2}{\beta\tau_w} \end{bmatrix} \tag{2.28}$$

where

$$a_{21} = -\frac{1}{\beta^2 M}\left[\frac{v_o \text{ Cos } \delta_o \ \Psi_{fo}}{X_d' \ \tau_{do}'} + \frac{(x_d' - x_q)}{X_d' \ X_q} v_o^2 \text{ Cos } 2\delta_o\right]$$

$$a_{23} = -\frac{v_o \text{ Sin } \delta_o}{\beta^2 M \ X_d' \ \tau_{do}'}$$

$$a_{31} = -\frac{(x_d - x_d')}{\beta \ X_d'} v_o \text{ Sin } \delta_o$$

$$a_{33} = -\frac{X_d}{\beta \ X_d' \ \tau_{do}'}$$

$$a_{41} = - \frac{\mu_e v_o}{\beta \tau_e v_{to}} \left[\frac{x_q v_{do} \operatorname{Cos} \delta_o}{X_q} - \frac{x_d' v_{qo} \operatorname{Sin} \delta_o}{X_d'} \right]$$

$$a_{43} = - \frac{\mu_e}{\beta \tau_e v_{to}} \frac{x v_{qo}}{X_d' \tau_{do}'}$$

$$(2.29)$$

M = inertia constant

τ_e = exciter time constant

τ_g = gate time constant

τ_w = water time constant

μ_e = exciter gain

σ = permanent droop.

The control matrix

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T \qquad (2.30)$$

The relevant data for the power system considered above are, [6]:

Transmission Line

$x = 0.7417, \quad B = 0.1339$

Synchronous Machine

$x_d = 1.00, \; x_d' = 0.27, \; x_q = 0.60, \; \tau_{do}' = 9.00,$

$M = 0.2122, \; D = 0.00537.$

Exciter and Voltage Regulator System

$\mu_e = 10.00, \; \tau_e = 1.0, \; \tau_s = 0.50.$

Governor and Hydraulic System

$\sigma = 0.045, \quad \tau_g = 0.10, \ \tau_a = 0.01, \ \tau_w = 1.60.$

The numerical values of μ_s and μ_a are not needed as they are included in u_v and u_g, respectively.

Nominal Operating Conditions :

$P_o = 0.735$ p.u., $Q_o = 0.034$ p.u., $v_{to} = 1.05$ p.u.

The initial currents, voltages, flux linkages and torque angle are:

$i_{do} = 0.286, \ i_{qo} = 0.640, \ v_{do} = 0.384, \ v_{qo} = 0.977,$

$v_f = 1.263, \ v_o = 1.058, \ \Psi_{fo} = 9.491, \ \delta_o = 0.887.$

The time is scaled in computation by a factor $\beta = 7.308$.

The numerical value of the system matrix A (eqn.(2.30)) is

$$A = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.683 & -0.0346 & -0.0816 & 0.0 & 0.0 & 0.882 & 0.0 & 1.32 \\ -0.0832 & 0.0 & -0.0254 & 0.137 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.130 & 0.0 & -0.107 & -0.137 & -1.37 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.274 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.0265 & 0.0 & 0.0 & 0.0 & -0.0616 & -1.37 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -13.70 & 0.0 \\ 0.0 & 0.0531 & 0.0 & 0.0 & 0.0 & 0.123 & 2.74 & -0.171 \end{bmatrix} \tag{2.31}$$

2.2.2 Power System Considered by Davison and Rau [21]:

The power system considered in this section (Figure 2.4) differs from the system of Section 2.2.1. The synchronous generator is steam turbine driven and has one damper winding along each rotor circuit. The generator is connected to an infinite bus via a short line whose resistance is small and hence represented by an inductance only. Simplified representation for the governor turbine system is assumed. The control inputs considered are, u_1, the change in reference signal to the exciter and u_2, the change in valve position. The state variables considered are, flux linkages of the field winding Ψ_f, flux linkages of direct axis winding Ψ_d, flux linkages of direct axis damper winding Ψ_{kd}, flux linkages of quadrature axis winding Ψ_q, terminal voltage (measured) e_t, field voltage E_{fd}, mech. torque T_m, rotor speed ω and the load angle δ. Electrical transients in the electrical network, $p\Psi_d$, $p\Psi_q$ and armature resistances are neglected. The performance of the machine is represented by eqns. (2.32) to (2.43).

$$e_d = -\Psi_q \tag{2.32}$$

$$e_q = \Psi_d \tag{2.33}$$

$$\Psi_d = x_{ad}\, i_{fd} + x_{ad}\, i_{kd} - x_d\, i_d \tag{2.34}$$

$$\Psi_q = x_{aq}\, i_{kq} - x_q\, i_q \tag{2.35}$$

$$\Psi_{kd} = x_{ad}\, i_{fd} + x_{kkd}\, i_{kd} - x_{ad}\, i_d \tag{2.36}$$

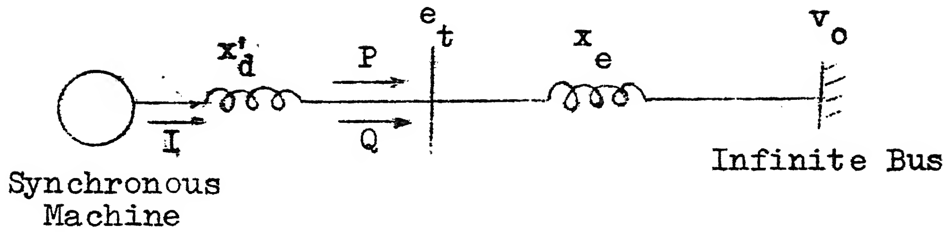

FIGURE 2.4 SYNCHRONOUS MACHINE INFINITE BUS SYSTEM

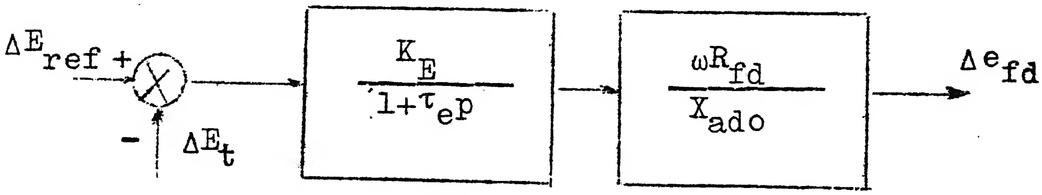

FIGURE 2.5 SIMPLIFIED EXCITER VOLTAGE REPRESENTATION

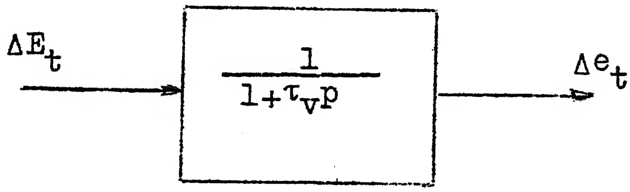

FIGURE 2.6 SIMPLIFIED VOLTAGE ERROR DETECTOR REPRESENTATION

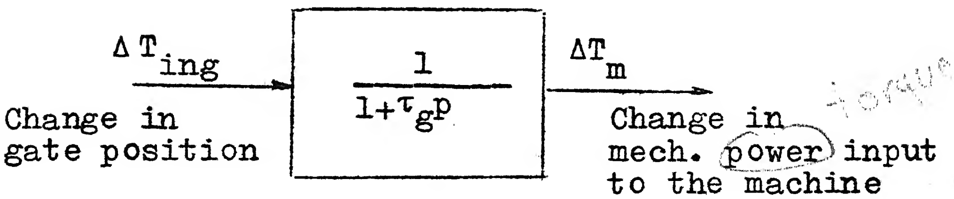

FIGURE 2.7 SIMPLIFIED POWER INPUT REPRESENTATION OF A THERMAL MACHINE

$$\Psi_{kq} = x_{kkq}\, i_{kq} - x_{aq}\, i_q \tag{2.37}$$

$$\overset{\circ}{\Psi}_{kd} = -\,\omega R_{kd}\, i_{kd} \tag{2.38}$$

$$\overset{\circ}{\Psi}_{kq} = -\,R_{kq}\, i_{kq} \tag{2.39}$$

$$e_{fd} = \overset{\circ}{\Psi}_{fd} + \omega R_{fd}\, i_{fd} \tag{2.40}$$

$$\Psi_{fd} = x_{ffd}\, i_{fd} + x_{ad}\, i_{kd} - x_{ad}\, i_d \tag{2.41}$$

$$\dot{\delta} = \omega \tag{2.42}$$

$$\overset{\circ}{\omega} = \frac{1}{2H}\left[-T_D\,\omega + \omega\, T_m - \Psi_d\, i_q + \Psi_q\, i_d\right] \tag{2.43}$$

On manupulating eqns. (2.32) to (2.43) (derivation omitted) the machine equations in the state variable form can be obtained.

A sixth order system is required to represent the machine while each of the exciter voltage system (Figure 2.5), voltage error detector (Figure 2.6) and the governor turbine system (Figure 2.7) is represented by a first order system. Thus a ninth order model for the power system is obtained. The machine terminal conditions give in d-q form, the following relation in linearized form:

$$\Delta I = [\Delta i_d\ \Delta i_q]^T$$

and

$$\Delta I = \begin{bmatrix} \frac{1}{x_e} & 0 & \frac{v_o \sin\delta_o}{x_e} \\ 0 & \frac{1}{x_e} & \frac{v_o \cos\delta_o}{x_e} \end{bmatrix} \begin{bmatrix} \Delta\Psi_d \\ \Delta\Psi_q \\ \Delta\delta \end{bmatrix} \tag{2.44}$$

The ninth order nonlinear equations representing machine, exciter voltage, voltage error detector and

governor turbine system are linearized around an operating condition and the linear model is obtained. For brevity the general form of the system matrix is not given here.

The relevant data for the power system considered in this section are [21]:

Transmission Line

$x_e = 0.25$

Synchronous Machine (Typical values of a 500 MW set)

$x_{ad} = 1.89$, $x'_d = 0.27$, $x''_d = 0.175$, $\tau''_{do} = 0.031$,

$\tau'_{do} = 4.3$, $\tau_{kd} = 0.0065$, $R_{kd} = 0.013$, $\tau_{kdo} = 0.392$,

$x_{aq} = 1.89$, $x''_q = 0.175$, $x_{fd} = 0.128$, $R_{fd} = 0.00124$,

$x = 0.15$, $x_q = 2.04$, $R_{kq} = 0.0261$, $\tau_{kqo} = 0.195$,

$v_o = 1.0$, $H = 6.75$, $D = 2.36$.

Exciter and Voltage Regulator System

$K_E = 155$, $\tau_e = 0.01$.

Voltage Error Detector System

$\tau_v = 0.01$

Governor and Turbine System

$\tau_g = 0.3$ sec,

The state vector $\underline{x}$ for the linear system is

$$\underline{x} = [\Delta\Psi_{fd}, \Delta\Psi_d, \Delta\Psi_{kd}, \Delta\Psi_q, \Delta e_t, \Delta E_{fd}, \Delta T_m, \Delta\omega, \Delta\delta]^T$$

and controls are ΔE_{ref} and ΔT_{ing}. The linearized system obtained in standard state space form as

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + D\,\underline{v} \tag{2.45}$$

where $\underline{v}$ is the disturbance vector. The numerical values of the matrices A, B and D are [21]:

$$A=\begin{bmatrix} -2.77 & -3.07 & 2.98 & 0.0 & 0.0 & 1.0 & 0.0 & 0.0 & -0.6 \\ 26.80 & -61.50 & 0.52 & 0.0 & 0.0 & 0.18 & 0.0 & -0.09 & -32.1 \\ 30.00 & -15.50 & -32.20 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -15.6 \\ 0.0 & 0.0 & 0.0 & -27.0 & -100.0 & 0.0 & 0.0 & -0.08 & -5.3 \\ 0.0 & 44.00 & 0.0 & -89.80 & -38.75 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -100.3 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -3.3 & 0.0 & 0.0 \\ 0.0 & -223.00 & 0.0 & -47.80 & 0.0 & 0.0 & 55.4 & -0.35 & -222.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} \tag{2.46}$$

$$B = D = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 25.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 3.33 & 0.0 & 0.0 \end{bmatrix}^T \tag{2.47}$$

$$\underline{u} = [\,\Delta E_{ref},\ \Delta T_{ing}]^T \tag{2.48}$$

and

$$\underline{v} = [\,\Delta E_{ref},\ \Delta T_{ing}]^T \tag{2.49}$$

2.3 DYNAMIC MODEL OF A TWO-AREA POWER SYSTEM [38]

Linear model of a two-area power system (Figure 2.8) is used in this thesis to design the load frequency

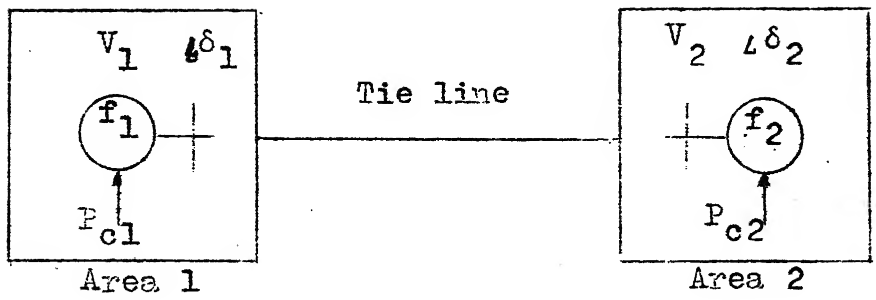

FIGURE 2.8 TWO AREA SYSTEM

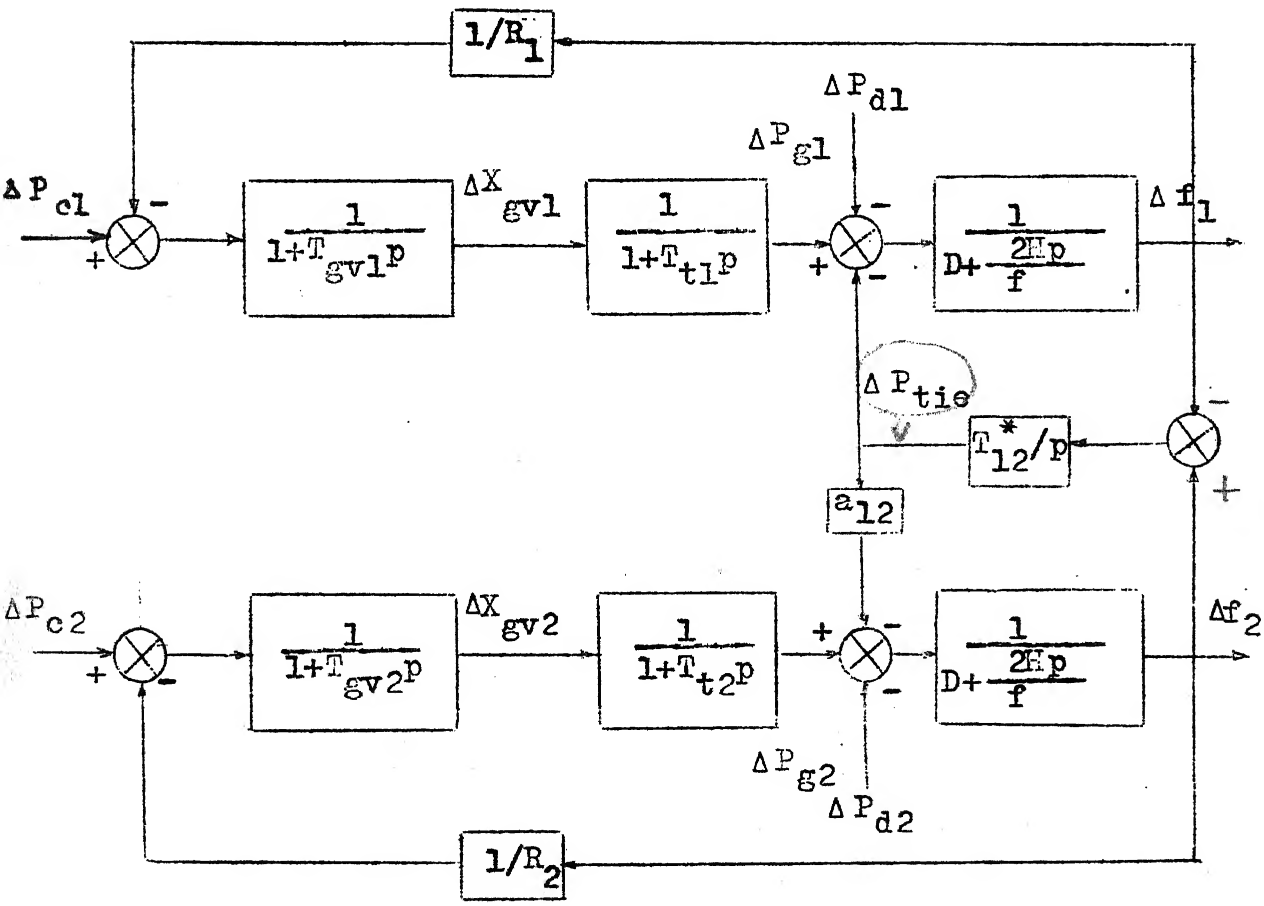

FIGURE 2.9 BLOCK DIAGRAM OF A 2-AREA LFC SYSTEM

controller. This model is the same as that considered by Fosha and Elgerd [38]. The function of the load frequency controller is to keep the frequencies f_1 and f_2 of the two areas and the tie line power flow P_{tie} at the desired level for all load conditions and to restrict their variations within a close tolerance. Furthermore, the steady state value of the area control error in each area should be zero. This is achieved by controlling the inputs P_{c1} and P_{c2} of the generators.

The linear model developed is based on the following assumptions:

(i) There is no interaction between the load frequency control channel and the voltage control channel and the node voltages are treated as constant during the analysis.

(ii) Strong electrical interconnection exists between generators of each area. Therefore, each area can be represented by an equivalent generator.

(iii) The two areas are interconnected by a weak tie line.

(iv) Losses are neglected.

(v) The system equations are linearized around an operating point.

(vi) The turbine, governor and generator of each area are represented by a first order system.

The state variables selected for the linearized model are

$$\underline{x} = [\int\Delta P_{tie}, \int\Delta f_1, \Delta f_1, \Delta P_{g1}, \Delta x_{gv1}, \int\Delta f_2, \Delta f_2, \Delta P_{g2}, \Delta x_{gv2}]^T \qquad (2.50)$$

The change in tie line power in the linearized form can be represented as

$$\Delta P_{tie} = T_{12}(\int\Delta f_1 \, dt - \int\Delta f_2 \, dt) \qquad (2.51)$$

where

$$T_{12} = \frac{2\pi}{x_{12}} \left| \frac{V_1 || V_2}{P} \right| \cos(\delta_1 - \delta_2)_o \qquad (2.52)$$

is a constant for the system defined as electrical stiffness of the line and is evaluated at the operating condition. The differential equations representing the system (see Figure 2.3) performance are:

$$p\Delta P_{tie} = T_{12}(\int\Delta f_1 \, dt - \int\Delta f_2 \, dt) \qquad (2.53)$$

$$\frac{2H}{f_o} p\Delta f_1 = - [D_1 \Delta f_1 + \Delta P_{tie} + \Delta P_{d1} - \Delta P_{g1}] \qquad (2.54)$$

$$p\Delta P_{g1} = - \frac{1}{T_{t1}} \Delta P_{g1} + \frac{1}{T_{t2}} \Delta X_{gv1} \qquad (2.55)$$

$$p\Delta X_{gv1} = - \frac{1}{T_{gv1}} \Delta X_{gv1} - \frac{1}{T_{gv1}R_1} \Delta f_1 + \frac{1}{T_{gv1}} \Delta P_{c1} \qquad (2.56)$$

$$p(\int\Delta f_1) = \Delta f_1 \qquad (2.57)$$

where f_o is the nominal frequency.

For each area 4 differential equations similar to eqns. (2.54) to (2.57) are required to represent the dynamic performance of the system for small variation in load. Thus a ninth order linear model obtained for a two area power system is

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \qquad (2.58)$$

For brevity the general form of the system matrix is not reported here. Also note that the origin of the state space of the above model is the steady state operating point.

The relevant data for the power system considered in this section are [38]:

Area 1:

Generator

$H_1 = 5.0$, $D_1 = 8.33 \times 10^{-3}$.

Governor-Turbine

$T_{t1} = 0.545$, $T_{gv1} = 0.08$, $R_1 = 2.4$.

Area 2:

Data for Area 2 are identical to those of Area 1.

Tie-Line:

$T_{12} = 0.545$

Operating Condition

$(\delta_1 - \delta_2)_o = 30^o$.

The non-zero elements of the system matrix A and control matrix B and disturbance matrix τ are:

$a_{1,2} = 0.545$, $a_{1,6} = -0.545$, $a_{2,3} = 1.0$, $a_{3,2} = -3.27$, $a_{3,3} = 0.05$, $a_{3,4} = 6.0$, $a_{3,6} = 3.27$, $a_{4,4} = -3.333$, $a_{4,5} = 3.333$, $a_{5,3} = -5.208$, $a_{5,5} = -12.5$, $a_{6,7} = 1.0$, $a_{7,2} = 3.27$, $a_{7,6} = -3.27$, $a_{7,7} = -0.05$, $a_{7,8} = 6.0$, $a_{8,8} = -3.333$, $a_{8,9} = 3.333$, $a_{9,7} = -5.2$, $a_{9,9} = -12.5$, $b_{5,1} = 12.5$, $b_{9,2} = 12.5$, $\tau_{3,1}=-6.0$ and $\tau_{7,2}=-6.0$.

2.4 CONCLUSION

In this chapter two models of power systems and one 2-area load frequency control model have been presented. These are used in subsequent chapters for illustrating the new modal control design techniques proposed in this thesis.

CHAPTER 3

OPTIMAL MODAL CONTROL OF POWER SYSTEMS

3.1 INTRODUCTION

In modern control theory there are mainly two approaches to the design of feedback controllers for linear, time-invariant dynamical systems to obtain satisfactory closed loop system behaviour: (1) procedures based on optimal control theory and (2) procedures based on pole assignment techniques.

Optimal linear regulator theory for linear, time-invariant systems with quadratic performance indices has been widely used, to design feedback controllers for power systems [6-8,20-46]. In this approach the state and control weighting matrices in the performance index are selected iteratively and the resulting sequence of optimal control problems are solved to obtain satisfactory closed-loop behaviour. Satisfactory behaviour of the closed loop system is checked either by studying the system response to certain test inputs [6-8,20,21,25], or by determining closed loop system poles [24]. The iterations involved in the optimal control theory approach are unattractive from the computational point of view.

As in the classical root locus procedure, locations of the closed loop system poles give the designer a 'feel' for the behaviour of the system. Thus, direct design procedures which assign closed loop system poles to

desirable locations are to be preferred to the optimal control theory approach mentioned above.

It is well known [66] that it is possible to achieve any arbitrary assigned closed loop eigenvalue locations (with the constraint that complex eigenvalues should occur in conjugate pairs), if the open loop system is controllable. In practice, however, the open loop eigenvalues which need to be shifted (that is, the dominant eigenvalues) are usually small in number. In such cases, it is not necessary that the system be completely controllable to obtain a feedback controller to effect the shift of the dominant eigenvalues: however, it is necessary that these eigenvalues be controllable. Modal control theory [52-55] offers a useful tool for designing controllers for such systems.

Modal analysis of dynamical systems is one of the best known and most extensively cultivated fields of classical physics. In the recent explosive development of control theory, the modal approach to the analysis and synthesis of multivariable control systems has led to many interesting and useful results [55]. The original adumbration of modal control theory was given by Rosenbrock [67]. Following Rosenbrock's work, many researchers developed various modal control procedures [52-55,68]. Some particular results relating to single input systems were obtained by Ellis and White [68]. The work of Simon and Mitter [52]

constitutes a major contribution to modal control theory. General closed form expressions for controller gains for single input systems were given by Porter et al [54-55] and Mayne et al [53].

In the power systems area, interest has been shown recently in direct pole assignment technique for controller design to obtain improved dynamic behaviour. Seraji [31] applied a pole assignment technique to a fifth order model of the synchronous generator system with a single input. Pai et al [49] considered a synchronous machine connected to an infinite bus with an improved representation of the exciter and the governor on the lines of Yu et al [6]. This resulted in an eighth order model of the power system with two control inputs. Applying modal control theory, they obtained feedback controllers and assigned dominant eigenvalues to desired closed loop pole locations. Their procedure involved utilizing only one of the available inputs to shift one or more eigenvalues at a time. This is a rather restrictive procedure and may result in high controller gains.

In this chapter, considering a dyadic structure, for the feedback matrix an algorithm is presented for designing an optimal modal controller; that is, a controller in which the feedback gains have been minimized in a particular sense. Computational difficulties

and problems in designing controllers for large systems are also indicated. The algorithm develcped is applied in designing control schemes for power systems, and the computational results are given.

3.2 DEVELOPMENT

Consider the linear, time-invariant, multi-input, multi-output system

$$\underline{\dot{x}} = A\,\underline{x} + B\,\underline{u} \tag{3.1}$$

Let the system eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ be distinct. It is desired to improve the system behaviour by shifting the 'ℓ' dominant eigenvalues, considered to be $\lambda_1, \lambda_2, \ldots, \lambda_\ell$ for convenience, by state feedback so that the closed loop system eigenvalues are $\{\rho_1, \rho_2, \ldots, \rho_\ell, \lambda_{\ell+1}, \lambda_{\ell+2}, \ldots, \lambda_n\}$. It is assumed that the dominant modes are controllable [52]. The general problem of assigning the dominant poles to new locations without disturbing the locations of the rest of the poles and without constraining the structure of the feedback matrix F in any manner is a computationally difficult problem, especially in large systems. The problem is simplified by assuming that the feedback law is of the type [52],

$$\underline{u} = F\,\underline{x} = \underline{\alpha}\,\underline{g}^T\,\underline{x} \tag{3.2}$$

where $\underline{\alpha}$ and $\underline{g}$ are m and n-vectors respectively. The pole assignment problem for (3.1) can be seen to be

equivalent to the pole-assignment problem for the single input system

$$\underline{x} = A\,\underline{x} + \underline{\sigma}\,z \tag{3.3}$$

where

$$\underline{\sigma} = B\,\underline{\alpha} \tag{3.4}$$

and $$z = \underline{g}^T\,\underline{x} \tag{3.5}$$

From (3.2) it can be seen that the elements α_i, $(i = 1,2,\ldots,m)$ of $\underline{\alpha}$ denote the relative proportions in which the m control inputs $u_1, u_2, \ldots, u_m$ are used. Let α_i's be so selected that the dominant modes of (3.3) are controllable [52]. It has been shown in Appendix A that for the desired pole assignment

$$\underline{g} = \sum_{i=1}^{\ell} k_i\,\underline{v}_i \tag{3.6}$$

where $\underline{v}_i$, $(i = 1,2,\ldots,\ell)$ are the reciprocal eigen-vectors of system matrix A associated with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_\ell$ respectively, the modal controller gains k_i, $(i = 1,2,\ldots,\ell)$ are

$$k_i = [\prod_{j=1}^{\ell}(\rho_j - \lambda_i)]/[\underline{v}_i^T\,B\,\underline{\alpha}\prod_{\substack{j=1\\ j\neq i}}^{\ell}(\lambda_j - \lambda_i)] \tag{3.7}$$

If we assume that in the closed loop system, complex eigenvalues appear in conjugate pairs, the vector $\underline{g}$ is then real [Appendix A].

3.3 OPTIMAL MODAL CONTROL

The feedback law (eqn.3.2) can be realized optimally so that the feedback gains, that is the elements of the modal controller matrix F are minimized. This may be done by choosing α_i, $(i = 1,2,\ldots,m)$ to minimize the performance index

$$J_1 = \sum_{j=1}^{m} \sum_{i=1}^{n} \alpha_j^2 \, g_i^2 \qquad (3.8)$$

where the elements g_i, $(i = 1,2,\ldots,n)$ are given by eqns.(3.6) and (3.7). Since for a given $\underline{\alpha}$ and $\underline{g}$, $\underline{\alpha}\,\underline{g}^T = (r\,\underline{\alpha})(\frac{1}{r}\,\underline{g}^T)$, where r is any scalar, we may impose the constraints

$$-1 \leq \alpha_j \leq +1 \ , \ (j = 1,2,\ldots,m) \qquad (3.9)$$

for computation convenience. Instead of choosing the objective function of eqn.(3.8) with constraints (eqn.3.9) we may choose the following simpler objective function for minimization without any constraints:

$$J_2 = \sum_{i=1}^{m} \alpha_i^2 + \sum_{j=1}^{n} g_j^2 \qquad (3.10)$$

Though the feedback gains are not directly minimized in the above objective function, it is clear from eqn.(3.10) that minimizing J_2 should yield small values for the feedback gains $\alpha_j \, g_i$, $(j= 1,2,\ldots,m),(i=1,2,\ldots,n)$. It is not necessary to impose any magnitude constraints such as in eqn.(3.9). Any of the well known function

minimization techniques (see, for example [69]), may be used to solve either of the minimization problems enunciated above. Computational results reported in this thesis have been obtained by using the gradient method described in Appendix B. The method of Grad and Brebner [70] based on the algorithm proposed by Wilkinson [71] is used for computing the eigenvalues and eigenvectors.

3.4 ALGORITHM TO REALIZE THE OPTIMAL MODAL CONTROLLER

i) Set iteration count n = 1.

ii) Set objective function value $J^{(n)} = 0$, where the objective function J is of the form (eqn.3.8 or eqn.3.10) and superscript n denotes the iteration count.

iii) Determine the dominant eigenvalues and the corresponding reciprocal eigenvectors of the system matrix A.

iv) Select new locations for the dominant eigenvalues.

v) Generate $\underline{\sigma}$ from eqn.(3.4) by choosing $\underline{\alpha}$ such that the mode controllability condition, $\underline{v}_j^T \, \underline{\sigma} \neq 0$, $(j=1,2,\ldots,\ell)$ is satisfied.

vi) Calculate k_j, $(j=1,2,\ldots,\ell)$ from eqn.(3.7) and vector $\underline{g}$ from eqn.(3.6).

vii) Calculate the new objective function value $J^{(n+1)}$ from eqn.(3.8) or eqn.(3.10) as the case may be.

viii) If $|J^{(n+1)}-J^{(n)}|$ is less than a specified small positive number, go to step (xi).

ix) Change $\underline{\alpha}$ vector as $\underline{\alpha}_{new} = \underline{\alpha} + c(\Delta\underline{\alpha})$ where c is step length and $(\Delta\underline{\alpha})$ is the negative of the gradient of the Lagrangian function with respect to $\underline{\alpha}$ [see Appendix B, eqns.(B.7) or (B.13)]. Check whether the mode controllability criterion for dominant modes is satisfied by the new $\underline{\alpha}$ vector. If it is not satisfied, then change c appropriately to satisfy the criterion.

x) Increment the iteration count n by 1. Go to step (vi).

xi) Calculate the modal controller matrix F from eqn.(3.2).

xii) Stop.

3.5 THE CONTROL STRUCTURE

The general structure of the closed loop control for a multi-input, multi-output (MIMO) system is shown in Figure 3.1. The state-variables are continuously monitored via sensors. The operation of the control system is based on signals which represent deviation of the states from the nominal settings. If the linear model is considered, then the variables to be considered are incremental quantities with respect to the operating point. Thus the models discussed in Chapter 2, Sections 2.2 and 2.3 can be interpreted in terms of the block diagram (Figure 3.1) appropriately. To illustrate application of the optimal modal controller algorithm of Secs.3.3&3.4 , two power systems problems are considered in this chapter:

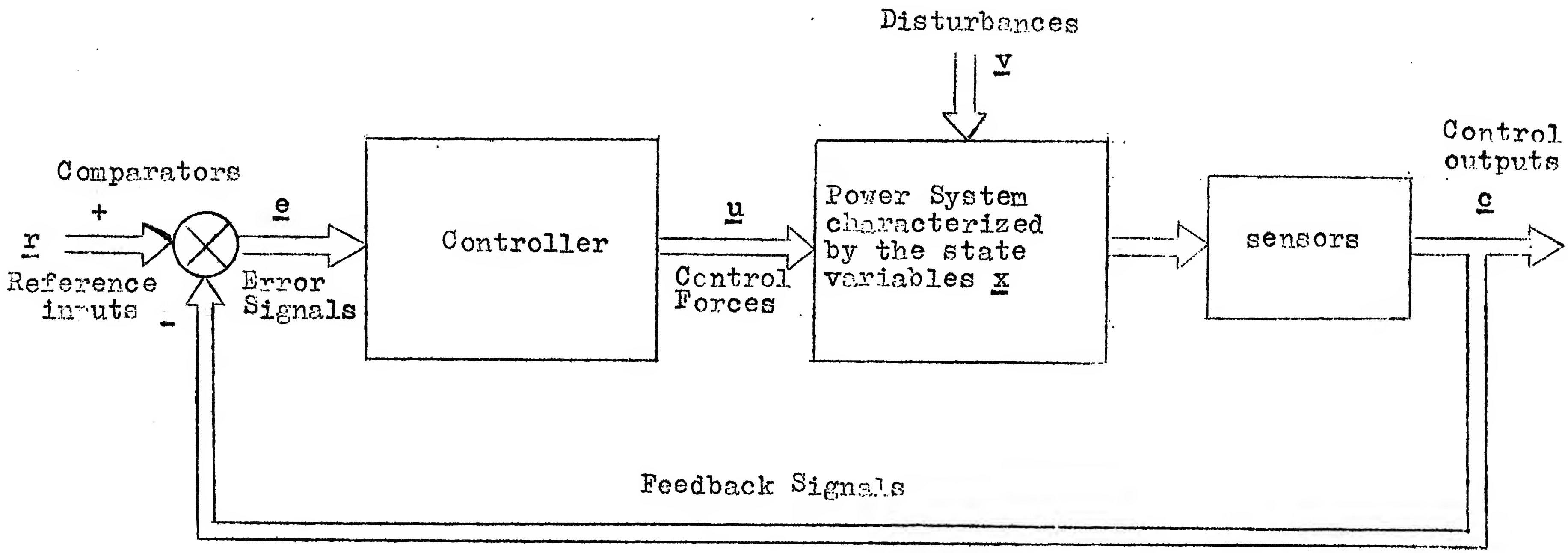

FIGURE 3.1 STRUCTURE OF MULTIPLE-INPUT MULTIPLE-OUTPUT CONTROL SYSTEM

(a) Controller design to improve dynamic response of a power system, and

(b) Load frequency control problem for a two-area power system.

3.6 DESIGN OF MODAL CONTROLLERS FOR POWER SYSTEMS

In this section, we shall apply the theory of Sections 3.3 and 3.4 to improve the dynamic performance of a power system. Two systems whose linear models were presented in Chapter 2 are considered.

(1) Synchronous machine connected to an infinite bus and driven by hydraulic turbine.

(2) Multi-area load frequency system.

3.6.1 Synchronous Machine-Infinite Bus system:

The power system considered consists of a synchronous generator connected to an infinite bus through along transmission line. The generator is driven by a hydraulic turbine. The system has an exciter voltage regulator and a turbine speed governor. The generator dynamics are represented by a third order system, while the exciter voltage regulator and primemover-speed governor dynamics are represented by second order and third order system respectively. This power system was considered by Yu et al [6] for designing an optimal regulator while Pai et al [49] designed a modal controller for this system. The mathematical model of the system has been discussed in Section 2.2.1. The state space model is given by

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u}$$

where A and B are given by eqns. (2.28) and (2.30) respectively. The operating points correspond to 0.735 p.u. real power, 0.034 p.u. reactive power delivered by the machine and 1.05 p.u. terminal voltage. The eigenvalues of the uncontrolled system matrix A are

$$-0.0114 \pm j0.7986; \; -0.0572; \; -0.0772 \pm j0.1146;$$
$$-0.1952; \; -0.2740; \; -13.70.$$

From these eigenvalues it can be inferred that though the uncontrolled system is stable, the response will be unsatisfactory due to the locations of the first five eigenvalues very near to the $j\omega$ axis. These eigenvalues are termed as 'Dominant eigenvalues'. The uncontrolled system requires considerable time to settle down to a steady state value when a disturbance takes place (see Figure 3.2). The disturbance considered is the release of a load perturbation ΔP_e which gives the initial conditions [49] as

$$\underline{x}(o) = [0.0258,\ 0.0,\ 0.0092,\ 0.0171,\ 0.0,\ 0.0,\ 0.0,\ 0.0]^T$$

The performance of the system is proposed to be improved by designing a modal control to shift the dominant eigenvalues to the following locations:

$$-0.40 \pm j0.915; \; -0.60; \; -0.96 \pm j0.720.$$

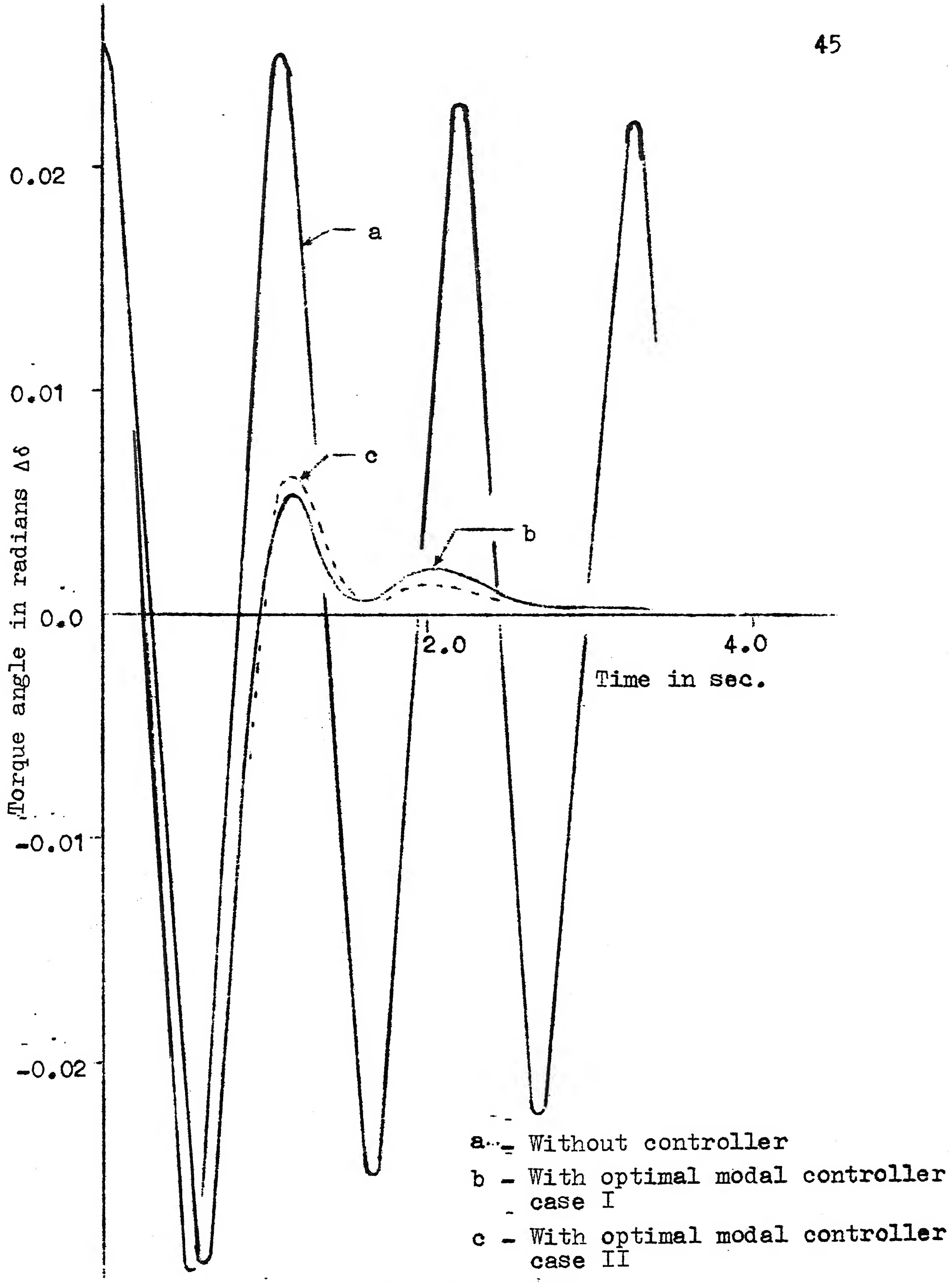

FIGURE 3.2 SYSTEM RESPONSE FOR Δδ

The locations of these eigenvalues has been justified in [49] on the basis of sector criterion.

The optimal modal controller algorithm developed in Section 3.4 is applied to design a modal controller for the above system. Lagrange multiplier formulation and gradient technique are used for minimizing the cost function. The resulting modal controller matrices are as follows:

Case I:

$$F = \begin{bmatrix} -1.384 & -2.612 & 4.132 & -6.691 & 58.44 & -209.5 & 16.31 & -23.46 \\ 5.221 & 9.853 & -15.585 & 25.233 & -220.45 & 790.3 & -61.53 & 88.50 \end{bmatrix}$$

when the cost function J_1 is optimized (eqn. 3.8).

Case II:

$$F = \begin{bmatrix} -1.396 & -2.619 & 4.138 & -6.696 & 58.49 & -210.7 & 16.41 & -23.53 \\ 5.237 & 9.825 & -15.521 & 25.120 & -219.42 & 790.3 & -61.58 & 88.27 \end{bmatrix}$$

when the cost function J_2 is optimized (eqn. 3.10).

The norm of the above feedback matrices defined as $\sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}^2$ are 734019.63 and 734017.94 respectively.

The transient response characteristics of the power system with optimal controllers obtained are shown in Figure 3.2. The disturbance considered is the same as for the uncontrolled system. Standard 4th order Runge- Kutta method was used for solving the differential equations of the system. It is noted that the gain matrix F for both

the cases are almost identical. However, computationally minimization of J_2 is preferred since J_2 has a simpler form.

3.6.2 Load Frequency Control of a Two-Area LFC System:

The two area LFC system considered is the same as the one discussed by Elgerd and Fosha [38]. The two areas which are considered identical are connected via a tie-line. Each area is represented by an equivalent generator. A load frequency controller is to be designed to keep the frequencies in the two areas and the tie-line power flow at the desired level for all loads and to minimize their deviation during a disturbance. This is done by controlling the inputs to the machines in the respective area. The development of the 9th order state space model for the system has been presented in Section 2.3 where the numerical data are also given. The state space model is of the form

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u}\,.$$

The eigenvalues of the system matrix A are

0.0; 0.0; $-0.4972\pm j3.522$; $-1.296\pm j2.512$; -1.624; -13.26; -13.29.

The uncontrolled system is stable but requires a large ~~more~~ time to stabilize when a step load disturbance of 0.01 p.u. takes place in area one [37,38,42] . The performance of the system is to be improved by designing a modal controller to shift the dominant eigenvalues to the left, away from the $j\omega$-axis.

The set of dominant eigenvalues contains a pair of repetitive eigenvalues located at the origin. The algorithm developed in Section 3.5 cannot be applied to shift repetitive eigenvalues. This difficulty can be overcome either by having a reduced state space model [42] or by providing a suitable feedback which yields a set of dominant, distinct eigenvalues. This feedback should be such that the locations of the nondominant eigenvalues are not disturbed appreciably. The algorithm of Section 3.5 can then be applied to design a modal controller for this modified system. The resultant controller matrix is obtained by summing the controller feedback matrix for the modified system and the feedback matrix used for converting the repetitive eigenvalues into distinct eigenvalues. The latter approach has been used here in designing the load frequency controller.

Considering the first order eigenvalue sensitivity [56,72] and the structure of control matrix, it is found that the following feedback matrix

$$F_1 = \begin{bmatrix} 0. & -0.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

shifts the two repetitive eigenvalues of zero magnitude to two distinct locations. The eigenvalues of the modified system after this feedback are

-0.01017; $-0.1066 \pm j3.512$; -0.7537; $-0.7621 \pm j2.531$; -2.832; -13.15 ; -13.28.

The system matrix of the modified system is

$$\bar{A} = A + B F_1$$

Now an optimal modal controller feedback matrix 'F_2' is designed so that the eigenvalues of the resultant closed loop system are

$$-0.7537, \quad -2.0, \quad -2.18 \pm j4.201, \quad -2.7 \pm j3.629,$$
$$-2.832, \quad -13.15, \quad -13.28.$$

The locations of the two pairs of complex poles are the same as that obtained by Ramamoorty et al [42] and the location -2 is arbitrarily selected so that it lies well within the set of nondominant eigenvalues of matrix A. The other locations are the same as those obtained for matrix $\bar{A}$.

The algorithm of Section 3.4 is then applied to design an optimal modal controller to obtain the above closed loop spectrum. The optimal modal controller matrix F_2 is

<u>Case I</u>: Minimization of J_1.

$$F_2 = \begin{bmatrix} -3.0 & -2.22 & 0.26 & 1.73 & 0.52 & 3.18 & 1.48 & 1.69 & 0.38 \\ 10.53 & 7.75 & -0.89 & -6.05 & -1.81 & -11.10 & -5.17 & -5.91 & -1.32 \end{bmatrix}$$

and the resultant modal controller matrix is

$$F = \begin{bmatrix} -3.00 & -3.18 & 0.26 & 1.73 & 0.52 & 3.18 & 1.48 & 1.69 & 0.38 \\ 10.54 & 7.75 & -0.89 & -6.05 & -1.81 & -11.10 & -5.17 & -5.91 & -1.32 \end{bmatrix}$$

The norm of feedback matrix F defined as $\sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}^2$ is 435.

Case II: Minimization of J_2.

$$F_2 = \begin{bmatrix} -0.62 & -0.55 & 0.09 & 0.44 & 0.13 & 0.77 & 0.29 & 0.27 & 0.56 \\ 10.46 & 9.05 & -1.55 & -7.26 & -2.12 & -12.78 & -5.01 & -4.61 & -0.93 \end{bmatrix}$$

and the resultant feedback matrix is

$$F = \begin{bmatrix} -0.62 & -1.51 & 0.09 & 0.44 & 0.13 & 0.77 & 0.29 & 0.27 & 0.56 \\ 10.47 & 9.05 & -1.55 & -7.26 & -2.12 & -12.78 & -5.01 & -4.61 & -0.93 \end{bmatrix}$$

The norm of feedback matrix is 463.

The transient responses for the controlled system with above modal controllers are shown in Figures 3.3 and 3.4 for a step load disturbance of 0.01 p.u. in area one. The transient responses compare favourably with the responses given in reference [42] for the optimal controller. It is also noted that the two feedback matrices obtained above have nearly the same matrix norm.

3.7 CONCLUSION

A simple method for designing regulators based on modal control theory has been suggested to obtain improved performance of linear time-invariant systems. An efficient and straightforward algorithm to design optimal modal controller has been developed. The algorithm is applied to design suitable controllers for (a) improving dynamic response of a power system and (b) load frequency control of a two area system. The algorithm has also been applied to shift repetitive eigenvalues in the dominant set in the

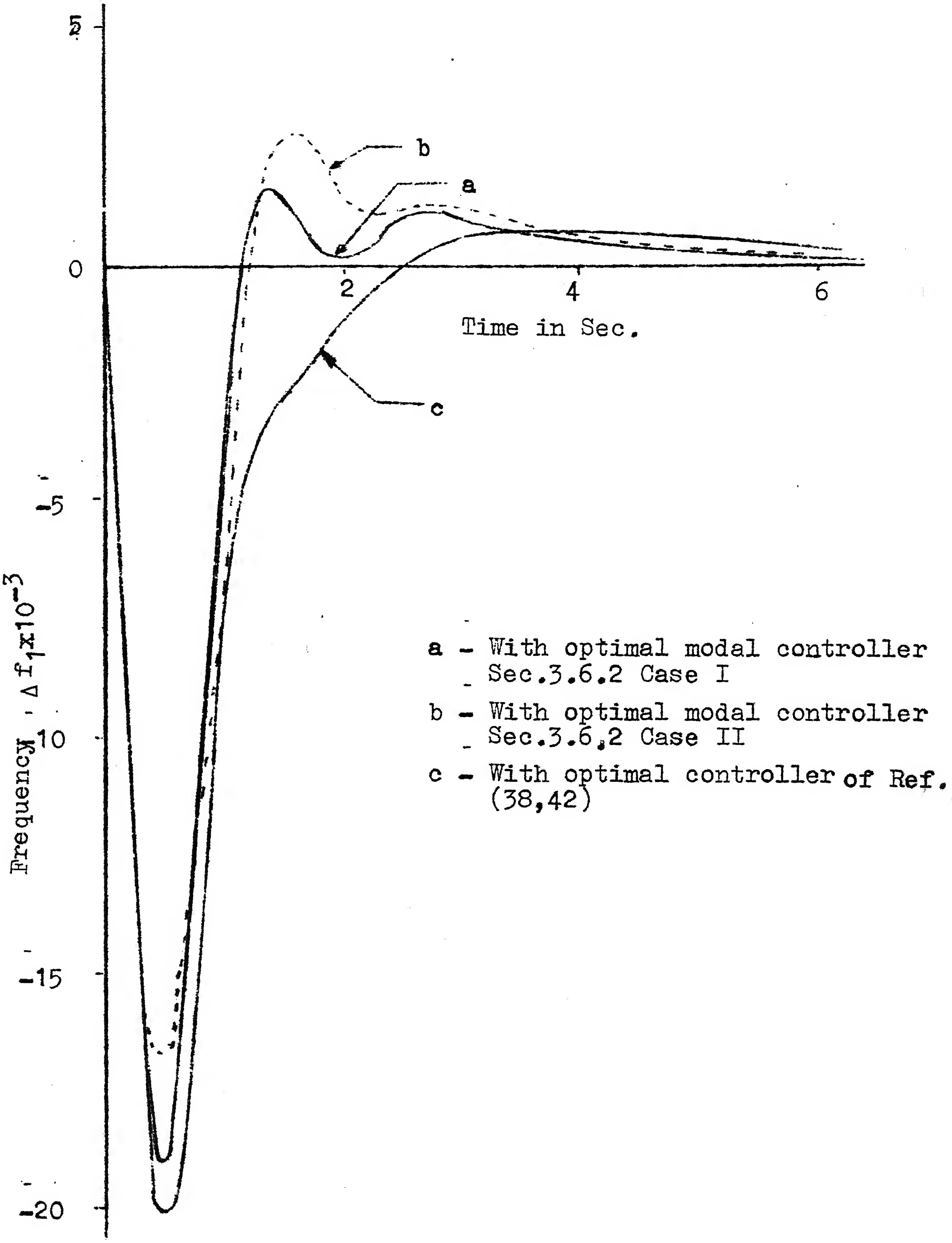

FIGURE 3.3 SYSTEM RESPONSE FOR Δf_1

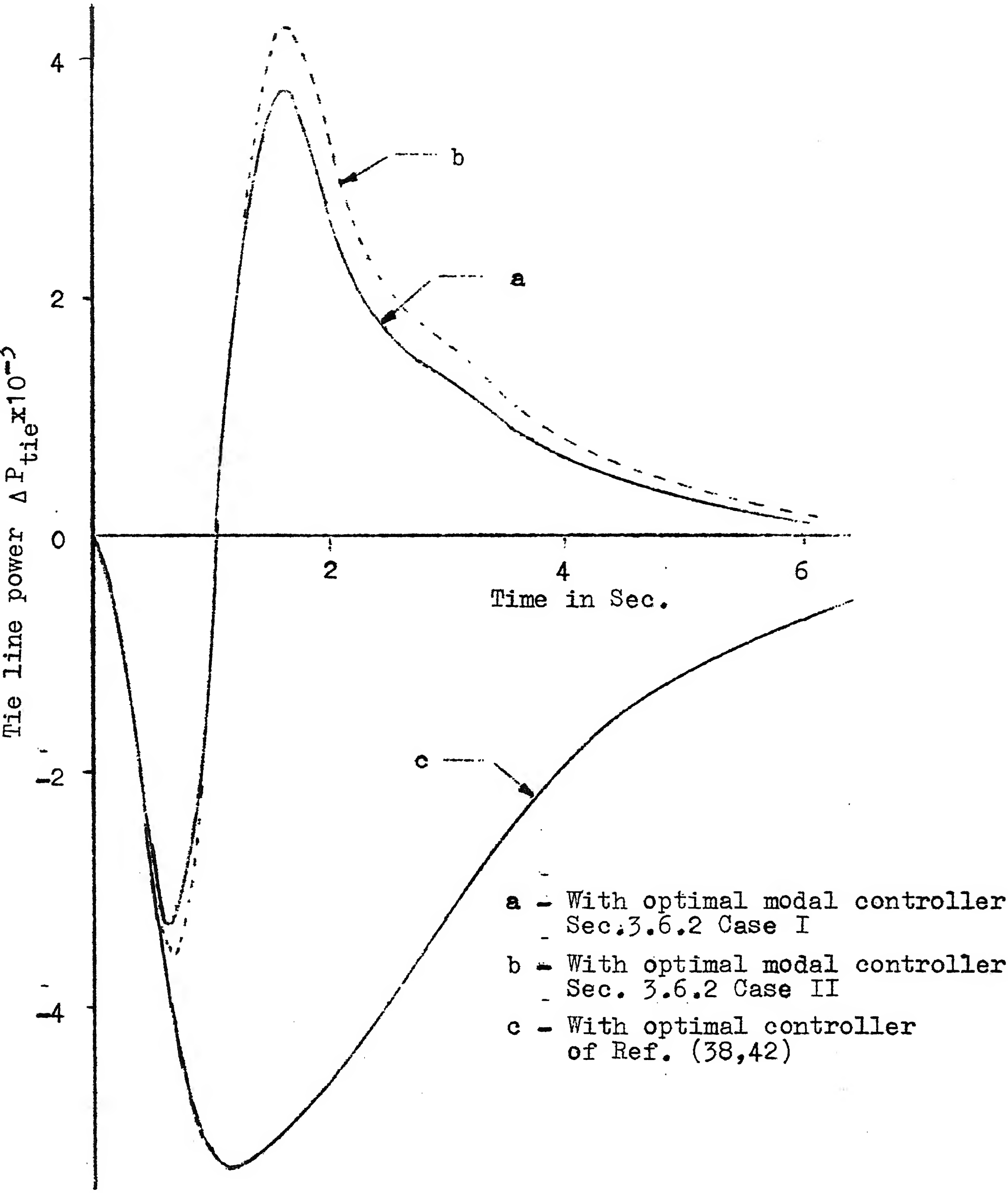

FIGURE 3.4 SYSTEM RESPONSE FOR ΔP_{tie}

case of load frequency control problem. In this case, the repititive eigenvalues are converted to distinct ones by a suitable feedback in order to enable application of the modal control algorithm.

Computational results show that in the problem considered, large feedback gains are required to shift all the dominant eigenvalues at one time to the desired closed loop locations. Large feedback gains are undesirable due to equipment limitations and noise amplification in the feedback path. It is,therefore, desirable to explore the possibilities of designing modal controllers with smaller gains. The problem of having reduced gains assumes more importance in designing controllers for large systems, where the number of dominant eigenvalues may be large. The dyadic structure for the controller matrix assumed in this chapter is rather too restrictive especially in high order systems with large number of inputs. In the next chapter we will discuss some simplified techniques for large dynamical systems.

CHAPTER 4

OPTIMAL MODAL CONTROLLERS FOR LARGE SYSTEMS

4.1 INTRODUCTION

The power utilities are generally interconnected to form large grids for economic and reliable operation. These interconnections result in large dimensions of the state and control vectors of the system models. From the computational point of view, the conventional feedback design techniques may not be suitable for these high order systems. Davison and Chadha [73] have established the advantages of the modal control approach for large systems.

The problem of developing suitable techniques for the design of controllers for large systems is an important and difficult one. Schemes based on modal reduction technique have been suggested for this problem [42, 74-76]. However, the closed loop system performance seems to be excessively sensitive to the reduced order model chosen and it may even be difficult to ensure satisfactory performance of the closed loop system [73, 74, 77]. It is desirable that, in addition to the technique developed being computationally attractive, it should also provide the designer with an insight into the problem which can enable him to utilize his engineering judgement to develop designs which are suitable from the practical view point. Modal control approach has this

feature [73]. Even though the approach of Davison and Chadha [73] is suitable for high order systems, the structure assumed for the controller is highly restrictive. This may result in rather large feedback gains. The same limitation exists in the approach given in Section 3.4. In this chapter, methods will be developed to overcome these limitations.

The simplicity of the approach of modal controller design developed in chapter 3 is essentially due to the dyadic structure assumed for the feedback matrix. This restriction on the structure of the feedback matrix also permits a simple procedure for optimizing feedback gains. As was mentioned in chapter 3, the general problem of determining optimal modal controllers in which no constraint is imposed on the structure of the feedback matrix is a computationally difficult one, especially for large systems. In the technique developed in this chapter, the modal controller feedback matrix is obtained as a sum of dyadic matrices. Thus the structure of the feedback matrix is more general than those assumed in chapter 3. This structure retains the simplicity and computational advantages of the dyadic structure in feedback optimization and yields lower feedback gains. It permits the designer to use his engineering judgement and intuition in developing suitable designs. In many situations, all the inputs need not have to be used for feedback control: some of them may be discarded without affecting the feedback gains appreciably.

The procedures of this chapter help the designer to identify the inputs to be used for feedback in such situations.

In essence, in the techniques reported in this chapter, the dominant eigenvalues are divided into groups and the algorithms of chapter 3 are applied sequentially to the various groups to obtain the required locations of closed loop poles. Thus for the pole allocation problem for any of the groups, the dyadic structure of the feedback matrix is retained. In many situations, dividing dominant eigenvalues into a small number of groups will yield structures significantly better than those obtained without grouping. It can thus be seen that algorithms being proposed are slightly more complex than those of chapter 3.

4.2 DEVELOPMENT OF METHOD BASED ON GROUPING

4.2.1 Eigenvalue Assignment in Groups

Consider the system defined by eqn. (3.1). Let the 'ℓ' dominant eigenvalues of the uncontrolled system be divided into 'p' groups as follows:

Group 1 : $[\lambda_1^{(1)}, \lambda_2^{(1)}, \ldots, \lambda_{\ell 1}^{(1)}]$

Group 2 : $[\lambda_1^{(2)}, \lambda_2^{(2)}, \ldots, \lambda_{\ell 2}^{(2)}]$

.

.

Group p : $[\lambda_1^{(p)}, \lambda_2^{(p)}, \ldots, \lambda_{\ell p}^{(p)}]$

where ℓ_q is the number of eigenvalues in q^{th} group.

Then, clearly $\ell = \ell_1 + \ell_2 + \ldots \ell_p$

It is assumed that each group has distinct eigenvalues. Let the corresponding desired locations for the closed loop poles be

$$\begin{array}{lll} \text{Group } 1 & : & [\rho_1^{(1)}, \rho_2^{(1)}, \ldots, \rho_{\ell 1}^{(1)}] \\ \text{Group } 2 & : & [\rho_1^{(2)}, \rho_2^{(2)}, \ldots, \rho_{\ell 2}^{(2)}] \\ \vdots & & \vdots \\ \text{Group } p & : & [\rho_1^{(p)}, \rho_2^{(p)}, \ldots, \rho_{\ell p}^{(p)}] \end{array}$$

In any of the groups above, complex eigenvalues appear in conjugate pairs.

Using eqn. (3.2), a dyadic matrix

$$F^{(1)} = \underline{\alpha}^{(1)} \, \underline{g}^{(1)\,T} \tag{4.1}$$

to shift the eigenvalues of Group 1 to their desired locations can be obtained. The resulting closed loop system matrix after shifting the eigenvalues in Group 1 is,

$$\begin{aligned} A^{(1)} &= A + B\,F^{(1)} \\ &= A + B\,\underline{\alpha}^{(1)} \, \underline{g}^{(1)\,T} \end{aligned} \tag{4.2}$$

Similarly a feedback matrix

$$F^{(2)} = \underline{\alpha}^{(2)} \, \underline{g}^{(2)T}$$

to shift the eigenvalues of Group 2 to their desired locations can be computed. However, in the computation, the reciprocal eigenvectors of the new system matrix $A^{(1)}$ given in eqn.(4.2),

corresponding to the eigenvalues of Group 2, are to be used. Thus in general, to shift the eigenvalues in the Group q after the assignment of the eigenvalues of the first (q-1) groups, the required feedback matrix is

$$F^{(q)} = \underline{\alpha}^{(q)}\ \underline{g}^{(q)T} \quad , \quad (q = 1,2,\ldots, p) \tag{4.3}$$

where

$$\underline{g}^{(q)} = \sum_{i=1}^{\ell_q} k_i^{(q)}\ \underline{v}_i^{(q)} \quad , \tag{4.4}$$

$$k_i^{(q)} = \Big[\prod_{j=1}^{\ell_q} (\rho_j^{(q)} - \lambda_i^{(q)})\Big] \Big/ \Big[\underline{v}_i^{(q)T}\ B\ \underline{\alpha}^{(q)} \prod_{\substack{j=1 \\ j\neq i}}^{\ell_q} (\lambda_j^{(q)} - \lambda_i^{(q)})\Big] \ ,$$

$$(i=1,\ \ldots,\ \ell_q) \tag{4.5}$$

$\underline{v}_i^{(q)}$, $(i=1,2,\ldots, \ell_q)$ denote the reciprocal eigenvectors of $A^{(q-1)}$ associated with eigenvalues $\rho_i^{(q)}$, $(i=1,2,\ldots, \ell_q)$ and the matrix $A^{(q)}$ is

$$A^{(q)} = A + \sum_{i=1}^{q} \underline{\alpha}^{(q)}\ \underline{g}^{(q)\ T} \tag{4.6}$$

The elements $\alpha_i^{(q)}$, $(i=1,2,\ \ldots,\ m)$ of $\underline{\alpha}$ denote the relative proportion in which the m control inputs are used in shifting the eigenvalues in the q^{th} group. These elements are to be so selected that the mode controllability condition

$\underline{v}_i^{(q)T}\ B\ \underline{\alpha}^{(q)} \neq 0$, $(i=1,2,\ \ldots,\ \ell_q)$ is satisfied

The resultant feedback matrix F is the sum of the p dyadic feedback matrices. Thus

$$F = \sum_{i=1}^{p} F^{(i)}$$

$$= \sum_{i=1}^{p} \underline{\alpha}^{(i)} \underline{g}^{(i)T} \qquad (4.7)$$

The rank of F is less than or equal to m.

The feedback law (4.7) can be realized optimally so that the feedback gains, that is elements of the modal controller matrix F are minimized. This may be done by choosing $\alpha_i^{(q)}$, $(i=1,2,\ldots,m)$, $(q=1,2,\ldots,p)$, to minimize

$$J = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\sum_{q=1}^{p} \alpha_j^{(q)} g_i^{(q)} \right)^2 \qquad (4.8)$$

The Lagrange multiplier technique may be used to solve the optimization problem as in Chapter 3.

4.2.2 Grouping Criterion

A criterion for grouping of dominant eigenvalues may be obtained from an examination of the equation which yields feedback gains for the case where the eigenvalues are not grouped [Chapter 3, eqns.(3.2), (3.6) and (3.7)]. The equations are reproduced below for convenience:

$$\underline{u} = F\,\underline{x} = \underline{\alpha}\,\underline{g}^T\,\underline{x} \qquad (3.2)$$

$$\underline{g} = \sum_{i=1}^{\ell} k_i\,\underline{v}_i \qquad (3.6)$$

$$k_i = \left[\prod_{i=1}^{\ell} (\rho_j - \lambda_i)\right] \Big/ \left[\underline{v}_i^T B\underline{\alpha} \prod_{\substack{j=1 \\ j\neq i}}^{\ell} (\lambda_j - \lambda_i)\right] \qquad (3.7)$$

$$(i=1,2,\ldots,\ell)$$

For given locations of eigenvalues, to obtain low feedback gains k_i, $\underline{v}_i^T B \underline{\alpha}$ should be large for $i=1,2,\ldots,\ell$ as seen from eqn. (3.7). In other words, the vector $\underline{\alpha}$ should be so chosen that the inner products of the vector $(B\underline{\alpha})$ with $\underline{v}_i$, $(i=1,2,\ldots,\ell)$ should be large. There is obviously a limitation to the gain reduction that can be achieved by arranging choice of $\underline{\alpha}$ in this manner.

Suppose now we group together those reciprocal eigenvectors (assumed to be normalized) such that product $\underline{v}_i^T \underline{v}_j$ is close to unity for each of the p groups. Then if the eigenvalues corresponding to any one of these p groups are moved by the algorithm of Chapter 3, the corresponding terms $\underline{v}_i^T B \underline{\alpha}$ will be uniformly large and the feedback gains correspondingly small. It is clear that, for any group, the vectors $B^T \underline{v}_i$ will be 'nearly' collinear.

From the arguments given above, it is clear that the mode controllability matrix

$$M = V_L^T B , \tag{4.9}$$

where the columns of the matrix V_L are the reciprocal eigenvectors of the system matrix A corresponding to the dominant eigenvalues, provides a suitable basis for grouping the eigenvalues. The eigenvalues corresponding to the 'nearly' linearly dependent rows of M are grouped together. For considering the linear dependency of the rows of matrix

M, the elements of the modal matrix M are replaced by their absolute values. The method is explained in terms of the numerical example in Section 4.4.

The feedback gains for the modal controller matrix will be generally less as the number of groups is increased. But the computational effort increases also with the increase in the number of groups. Therefore a compromise is to be made between the computational effort and the feedback gains while deciding upon the number of groups.

4.3 ALGORITHM TO REALIZE THE OPTIMAL MODAL CONTROLLER FOR LARGE SYSTEMS

We outline below the algorithm to move eigenvalues based on grouping technique.

i) Determine the dominant eigenvalues and the corresponding reciprocal eigenvectors of the system matrix A.

ii) Obtain the mode controllability matrix from eqn. (4.9). Divide the eigenvalues to be shifted into p groups using the criterion of Section 4.2.2. Arbitrarily order the groups.

iii) Let n denote the iteration count in the optimization process. Set n equal to 1.

iv) Set objective function $J^{(n)} = 0$.

v) Choose initial value of $\underline{\alpha}^{(1)}$ such that the modes of the first group are controllable.

vi) Set q equal to 1. 'q' denotes the group whose eigenvalues are being assigned.

vii) Compute $F^{(q)}$ from eqn. (4.3) through eqn.(4.4) and (4.5).

viii) Obtain the closed loop system matrix $A^{(q)}$ from eqn.(4.6).

ix) If q equal to p, then go to step(xiii).

x) If n=1, choose the initial values of $\underline{\alpha}^{(q+1)}$ such that the modes of $(q+1)^{th}$ group are controllable. Go to step(xii).

xi) Check whether the mode controllability criterion for dominant modes in $(q+1)^{th}$ group is satisfied. If the criterion is not satisfied then go to step(xvii).

xii) Increment the iteration count q by 1 and go to setp vii.

xiii) Calculate the new objective function $J^{(n+1)}$ from eqn. (4.8). If $|J^{(n+1)} - J^{(n)}|$ is less than a specified small positive number, then go to step(xviii).

xiv) Change $\underline{\alpha}^{(i)}$ vector as $\underline{\alpha}_{new}^{(i)} = \underline{\alpha}^{(i)} + c\ (\Delta\underline{\alpha}^{(i)})$, (i=1,2,...,p), where c is the step length and $(\Delta\underline{\alpha}^{(i)})$ is the negative gradient of the Lagrangian function with respect to $\underline{\alpha}^{(i)}$.

xv) Increment the iteration count n by 1.

xvi) Check, whether the eigenvalues of first group are controllable. If controllable then go to step(vi).

xvii) Change the step length c and compute $\underline{\alpha}^{(i)} = \underline{\alpha}_{old}^{(i)} + c\,(\Delta\,\underline{\alpha}^{(i)})$, $(i=1,2,\ldots,p)$. The value of c should be such that the modes of first group are controllable. Go to step(vi).

xviii) Obtain the modal controller matrix F from eqn.(4.7).

xix) Stop.

4.4 DESIGN OF OPTIMAL MODAL CONTROLLERS FOR POWER SYSTEMS BY EIGENVALUE GROUPING

Two examples of the design of optimal modal controllers by eigenvalue grouping are given in this section. One of them relates to the problem of improving the dynamic response of a power system, while the other pertains to the two area load frequency problem.

4.4.1 Example 1: Power System

The power system considered is the same as in Section 3.6.1. The desired closed loop pole locations are the same as those considered in Section 3.6.1.

The mode controllability matrix defined by eqn.(4.9) is

$$M = \begin{bmatrix} -1.7\text{x}10^{-2} - j5.2\text{x}10^{-3} & 1.32\text{x}10^{-1} + j1.67\text{x}10^{-2} \\ -1.7\text{x}10^{-2} + j5.2\text{x}10^{-3} & 1.32\text{x}10^{-1} - j1.67\text{x}10^{-2} \\ -1.1\text{x}10^{-2} & 0.96\text{x}10^{-1} \\ 0.99+j0.0 & -5.0\text{x}10^{-2} + j4.05\text{x}10^{-3} \\ 0.99-j0.0 & -5.0\text{x}10^{-2} - j4.05\text{x}10^{-3} \end{bmatrix}$$

It is observed that

$$\frac{|M_{11}|}{|M_{12}|} = \frac{|M_{21}|}{|M_{22}|} \simeq \frac{|M_{31}|}{|M_{32}|}$$

and

$$\frac{|M_{42}|}{|M_{41}|} = \frac{|M_{52}|}{|M_{51}|}$$

where m_{ij} is the element of matrix M corresponding to i^{th} row and j^{th} column. Therefore rows 1,2 and 3 may be considered nearly linearly dependent. Using the grouping criteria of Section 4.2.2, the dominant eigenvalues are divided into the two following groups.

Group I : $[-0.0115 \pm j0.915, -0.60]$

Group II : $[-0.96 \pm j0.720]$

The algorithm detailed in Section 4.3 is used and the cost function defined in eqn. (4.8) is optimized. The optimal modal control feedback matrix F is computed to be

$$F = \begin{bmatrix} -0.474 & -0.733 & 1.910 & 1.312 & -1.853 & -1.996 & -0.053 & -1.289 \\ 0.294 & -5.250 & 2.094 & 1.296 & -1.794 & 5.328 & -1.232 & -3.765 \end{bmatrix}$$

Let the norm of the matrix F = $\sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}^{2}$ (4.10)

where f_{ij} is the element of matrix F corresponding to i^{th} row and j^{th} column.

The norm of above matrix F is 96.22. Thus it can be seen that considerable reduction in feedback gains is obtained

by assigning the eigenvalues in groups (see col.1,2 and 3 of Table 4.1). The response of the power system with this modal controller matrix is better than the modal controller feedback matrix obtained by shifting all the eigenvalues in one group (see Fig.4.1 and Fig.3.2).

4.4.2 Example 2 : L.F.C Problem

A two area power system is considered for designing modal controller for load frequency control of the system. This problem was also considered in Section 3.6.2.

The dominant eigenvalues are divided into two groups as mentioned below.

Group I : $[-0.102, -0.1066 \pm j3.512]$

Group II : $[-0.762 \pm j2.5310]$

The modal controller feedback matrix F, obtained by utilizing the algorithm of Section 4.3 and considering the cost function defined in eqn. (4.8) is

$$F = \begin{bmatrix} -0.468 & -2.477 & -1.960 & -2.024 & -0.406 & -0.749 & -0.089 & 0.304 & 0.112 \\ 3.204 & 1.245 & 1.154 & 0.861 & 0.132 & -1.226 & -1.054 & -1.604 & -0.396 \end{bmatrix}$$

The norm of the feedback matrix F as defined in eqn.(4.10) is 29.57.

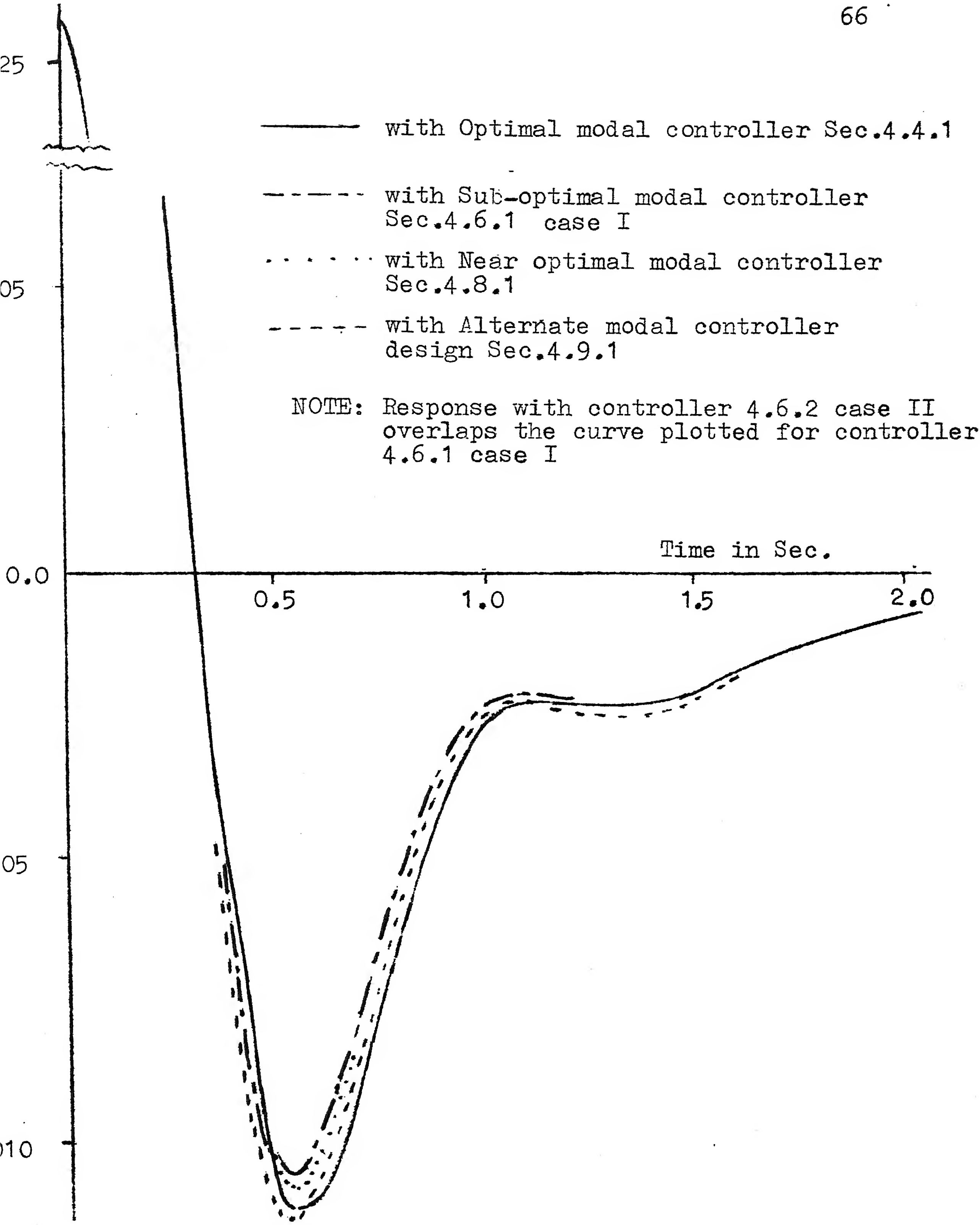

FIGURE 4.1 SYSTEM RESPONSE FOR $\Delta\delta$

TABLE 4.1 : COMPARISON OF FEEDBACK MATRICES OBTAINED BY VARIOUS DESIGN PROCEDURES - POWER SYSTEM (Sec.2.2.1)

Type of Design	Optimal Design without grouping Sec.3.6.1		Optimal design with grouping Sec.4.4.1	Suboptimal design with grouping Sec.4.6.1		Near optimal design Sec.4.8.1	Alternate design Sec.4.9.1
	J_1	J_2	J_1	J_1	J_2		J_3
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Value of J	734019.6	734017.94	96.22	233.23	236.07	164.2	117.2

TABLE 4.2 : COMPARISON OF FEEDBACK MATRICES OBTAINED BY VARIOUS DESIGN PROCEDURES - L.F.C. PROBLEM (Sec.2.3)

Type of Design	Optimal Design without grouping Sec.3.6.2		Optimal design Sec.4.4.2	Suboptimal design Sec.4.6.2
	J_1	J_2	J_1	J_2
	(1)	(2)	(3)	(4)
Value of J	435	463	29.57	86.82

The feedback gain matrix when eigenvalues are shifted in groups has a smaller norm than that obtained for the case when eigenvalues are shifted in one group only. Response of the closed loop system with the initial condition of Section 3.6.2 is given in Fig.4.2 and Fig.4.3.

4.5 SUB-OPTIMAL MODAL CONTROLLER

The algorithm detailed in the previous section yields small feedback gains for large values of p where p denotes the number of groups. However, more computational effort will be required in solving the optimization problem because in the iteration process for shifting the eigenvalues in any group, the reciprocal eigenvectors associated with the eigenvalues in that group are to be computed. Therefore for large systems, where the dominant eigenvalues may have to be divided into large number of groups, the above procedure may not be computationally attractive. For such systems a procedure of designing sub-optimal modal controllers which needs smaller computational effort is presented in this section.

The dominant eigenvalues are divided into p groups. Instead of optimizing the resultant feedback matrix F after shifting all the eigenvalues, the optimization procedure is carried out in stages i.e. the feedback matrix required to shift the eigenvalues in each group is optimized. The resultant

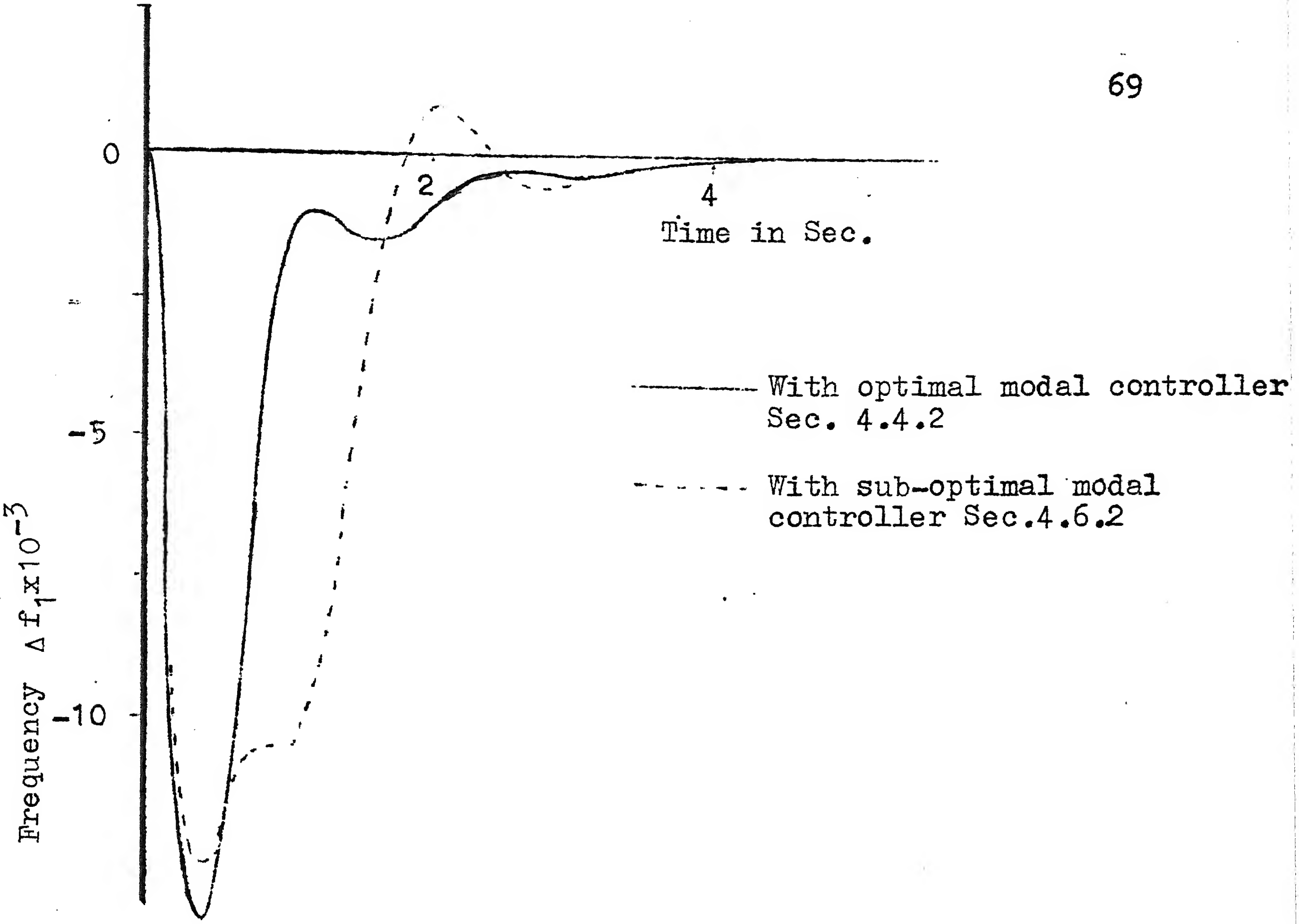

FIGURE 4.2 SYSTEM RESPONSE FOR Δf_1

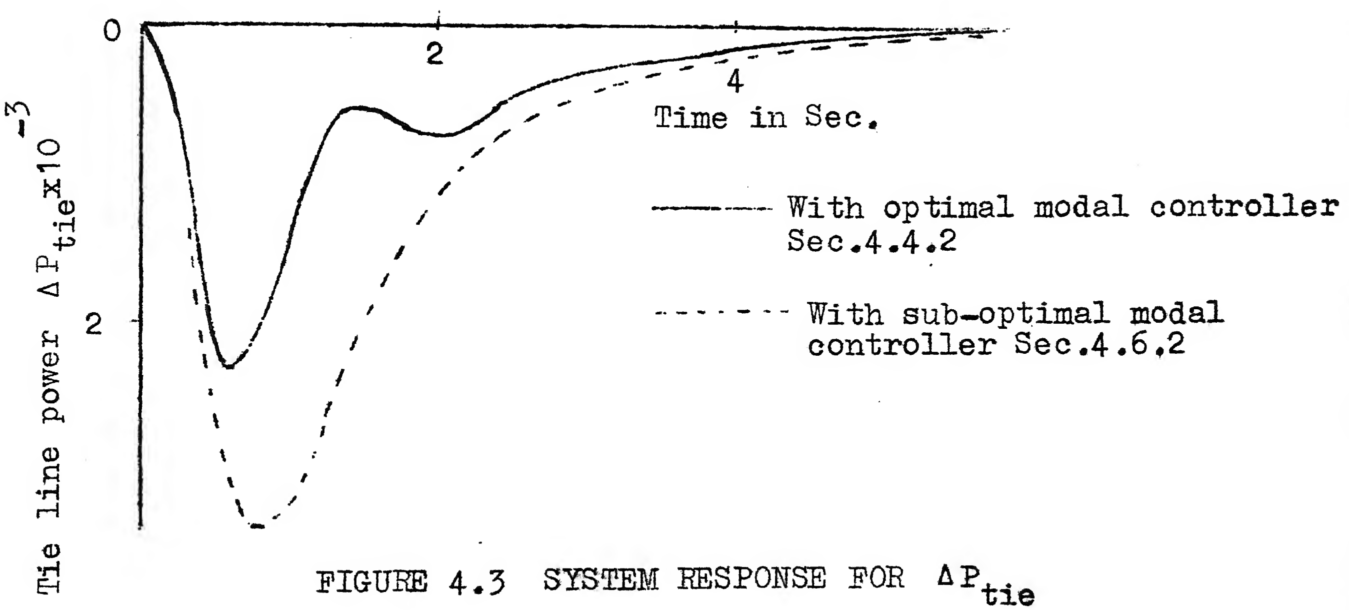

FIGURE 4.3 SYSTEM RESPONSE FOR ΔP_{tie}

feedback matrix is determined by summing the p feedback matrices thus obtained, that is,

$$F = \sum_{i=1}^{p} F_i \qquad (4.11)$$

where F_i is the optimum feedback matrix to shift the eigenvalues of i^{th} group to the desired locations with eigenvalues of the first (i-1) groups already assigned. In this approach the reciprocal eigenvectors are computed only for a total of p times. Either of the performance indices defined in eqns.(3.8) and (3.10) may be selected for the optimization problem.

4.6 DESIGN OF SUBOPTIMAL MODAL CONTROLLERS FOR A POWER SYSTEM

Two examples of the design of suboptimal modal controllers are considered in this section to demonstrate the simplicity of the method explained in the above section.

4.6.1 Example 1

The system of Section 4.4.1 is considered for designing suboptimal modal controller. The distribution of eigenvalues in groups and their desired closed loop locations are identical to those in Section 4.4.1. The feedback matrix F defined in eqn.(4.11) is computed as detailed above.

COMPUTATIONAL RESULTS

Case I: Optimization of J_1 (eqn.(3.8))

$$\underline{\alpha}^{(1)} = [-0.4501 \quad 4,118]^T$$

$$\underline{\alpha}^{(2)} = [\ 4.085 \quad 0,5714]^T$$

$$F = \begin{bmatrix} 1.297 & 0.998 & 1.924 & 1.226 & -1.707 & -10.920 & 0.224 & -4.234 \\ 1.053 & -4.895 & 0.353 & 0.071 & -0.068 & 6.889 & -1.289 & -3.253 \end{bmatrix}$$

The norm of the matrix F defined by eqn. (4.10) is 233.23

Case II: When the cost function J_2 defined in eqn.(3.10) is optimized:

$$\underline{\alpha}^{(1)} = [-0.3542 \quad 3.176]^T$$
$$\underline{\alpha}^{(2)} = [\ 3.384 \quad 0.4562]^T$$

$$F = \begin{bmatrix} 1.314 & 1.014 & 1.927 & 1.227 & -1.780 & -11.015 & 0.226 & -4.266 \\ 1.048 & -4.895 & 0.344 & 0.065 & -0.059 & 6.928 & -1.289 & -3.234 \end{bmatrix}$$

The norm of the matrix F defined by eqn.(4.10) is 236.07.

The response of the closed loop control system with the sub optimal controllers design (see Fig.4.1) are comparable with the system response with optimal modal controller (Section 4.4.1) for identical disturbance and initial conditions. The norm of feedback matrices of the sub optimal designs is more than that of the optimal design.

4.6.2 Example 2:

The two area power system of Section 4.4.2 is considered for designing the suboptimal load frequency controller. The distribution of eigenvalues in groups and their desired closed loop locations are the same as in Section 4.4.2.

The cost function J_2 defined in eqn.(3.10) is minimized while computing the feedback matrix for each group. The resultant suboptimal modal controller feedback matrix is

$$F = \begin{bmatrix} 3.5079 & -1.4410 & -3.3881 & -3.9321 & -0.826 & -3.625 & 1.4417 & 1.7692 & 0.3481 \\ 3.8534 & 0.3214 & -1.1761 & -1.5190 & -0.334 & -2.9036 & -0.1427 & 0.1047 & 0.29 \end{bmatrix}$$

The norm of tthe matrix F defined by eqn.(4.10) is 88.83.

The response of the closed loop control system with suboptimal controller (see Fig.4.2 and 4.3) is comparable with the system response with optimal modal controller (Section 4.4.2). The norm of the suboptimal feedback matrix is higher than that of optimal feedback matrix.

4.7 NEAR OPTIMAL MODAL CONTROL WITH DOMINANT CONTROL INPUTS

Inspite of the improved technique suggested in Section 4.6 computational effort may still be considerable. This is due to the large dimension of the $\underline{\alpha}$ vector associated with each group of dominant eigenvalues with respect to which the objective function has to be minimized. However, the computational effort may be reduced by associating with each group of dominant eigenvalues a corresponding set consisting of a small number of inputs to be used for shifting the eigenvalues in the groups optimally. When

these inputs are properly selected, the objective function is marginally effected. Such inputs are termed as 'Dominant Control inputs'. These input sets may be formed by studying the 'degree of coupling' between the modes in a given group and the inputs. The degree of coupling between the i^{th} mode and j^{th} input may be considered to be given by

$$| \underline{v}_i^T \ \underline{b}_j |$$

where $\underline{v}_i$ is the normalized reciprocal eigenvectors of A associated with λ_i and b_j is the j^{th} column of matrix B. Let M_q be the mode controllability matrix associated with the q^{th} group, that is,

$$M_q = V_q^T B$$

where the columns of V_q are the normalized eigenvactors associated with the group. Then the elements of M_q indicate the degree of coupling of the eigenvalues in the group with the inputs. The inputs corresponding to the columns of M_q having relatively large magnitudes form the set of dominant inputs.

4.8 NEAR OPTIMAL MODAL CONTROL OF POWER SYSTEMS

The examples of the design of near optimal modal controllers are solved in this section to demonstrate the suitability of the method suggested in Section 4.7.

4.8.1 Example 1 :

The example of Section 4.4.1 is considered here with the same eigenvalues grouping.

The mode controllability matrices for the two groups are:

$$M_1 = \begin{bmatrix} -1.7\times10^{-2} - j5.2\times10^{-3} & 1.32\times10^{-1} + j1.67\times10^{-2} \\ -1.7\times10^{-2} + j5.2\times10^{-3} & 1.32\times10^{-1} - j1.67\times10^{-2} \\ -1.1\times10^{-2} & 0.96\times10^{-1} \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 0.99+j0.0 & -5.0\times10^{-2}+j4.5\times10^{-3} \\ 0.99-j0.0 & -5.0\times10^{-2}-j4.5\times10^{-3} \end{bmatrix}$$

On the basis of the degree of coupling (Section 4.7) between the modes in a group and the inputs it is found that the dominant control for Group I is u_2 and the dominant control for Group II is u_1. Since for shifting eigenvalues of each group only one dominant control input exists, there is a unique resultant feedback matrix for the pole assignment problem. The near-optimal modal controller feedback matrix is found to be

$$F = \begin{bmatrix} 0.481 & 0.184 & 1.818 & 1.204 & -1.766 & -6.566 & -0.093 & -2.774 \\ 0.881 & -5.028 & 0.086 & -0.100 & 0.183 & 8.380 & -1.320 & -2.678 \end{bmatrix}$$

The norm of the feedback matrix F as defined by eqn. (4.10) is 164.2. It is better than that of suboptimal design and is near to the value of the optimal feedback matrix (Section 4.4.1). Response of the closed loop system with near optimal modal controller for the disturbance

considered in Section 4.4.1 is plotted in Figure 4.1.

4.8.2 <u>Example 2:</u>

In this example the dynamic stability of steam turbine driven synchronous generator is improved by designing a near optimal modal controller.

The power system considered consists of a steam turbine driven synchronous generator connected to an infinite bus through a radial transmission line. The synchronous generator has damper winding along the direct as well as quadrature axes. Both the exciter and governor inputs are available as control inputs. This system was considered by Davison and Rau [21] for designing an optimal controller. The system and control matrices for the linearized model and the numerical values of parameters are given in Chapter 2 Section 2.2.2. The eigenvalues of the system matrix are $-0.101 \pm j10.47$, -3.33, -13.55, -29.62, $-33.867 \pm j10.239$, $-106.855 \pm j15.889$ and the desired dominant eigenvalues of the closed loop system are specified to be $-9.07 \pm j17.3$, -10.8, -17.3. These locations are the same as obtained by Davison and Rau [21]. The dominant eigenvalues of the uncontrolled system are divided into following two groups.

Group I : $[-0.1010 \pm j10.47, -3.33]$ and

Group II : $[-13.55]$

It is found that the dominant inputs for Group I and II are u_2 and u_1 respectively. The resultant feedback matrix of the modal controller is found to be

$$F = \begin{bmatrix} -14.2 & 6.4 & -2.04 & -42.24 & 6.5 & -0.149 & 1.6 & 0.44 & 4.66 \\ 8.7 & 7.5 & 0.36 & 2.41 & -2.4 & 0.081 & -7.6 & -2.50 & -4.74 \end{bmatrix}$$

The feedback matrix obtained by Davison and Rau [21] is

$$F = \begin{bmatrix} -1.45 & 0.14 & -0.26 & -1.94 & 2.62 & -0.14 & 0.52 & 0.005 & 0.41 \\ 11.40 & 7.62 & 0.60 & 4.50 & -1.98 & 0.07 & -8.64 & -2.770 & -9.44 \end{bmatrix}$$

It appears from the above that Davison and Rau's result is superior. However it must be noted that their feedback disturbs the position of the nondominant eigenvalues to such locations as to give better feedback gains. Responses of the closed loop system with near optimal modal controller are plotted in Figures 4.4 and 4.5.

4.9 AN ALTERNATE METHOD OF MODAL CONTROLLER DESIGN FOR LARGE SYSTEMS

Reciprocal eigenvectors are to be computed (p-1) times in each iteration of the optimization process described in Section 4.3 with the cost function J defined in eqn. (4.8), where p is the number of groups in which the dominant eigenvalues are grouped. This may require more computer time for each iteration, especially in the case of large systems. Therefore to decrease the computational time, either the number of groups should be small or suboptimal modal controller design procedure (Section 4.6) is to be used.

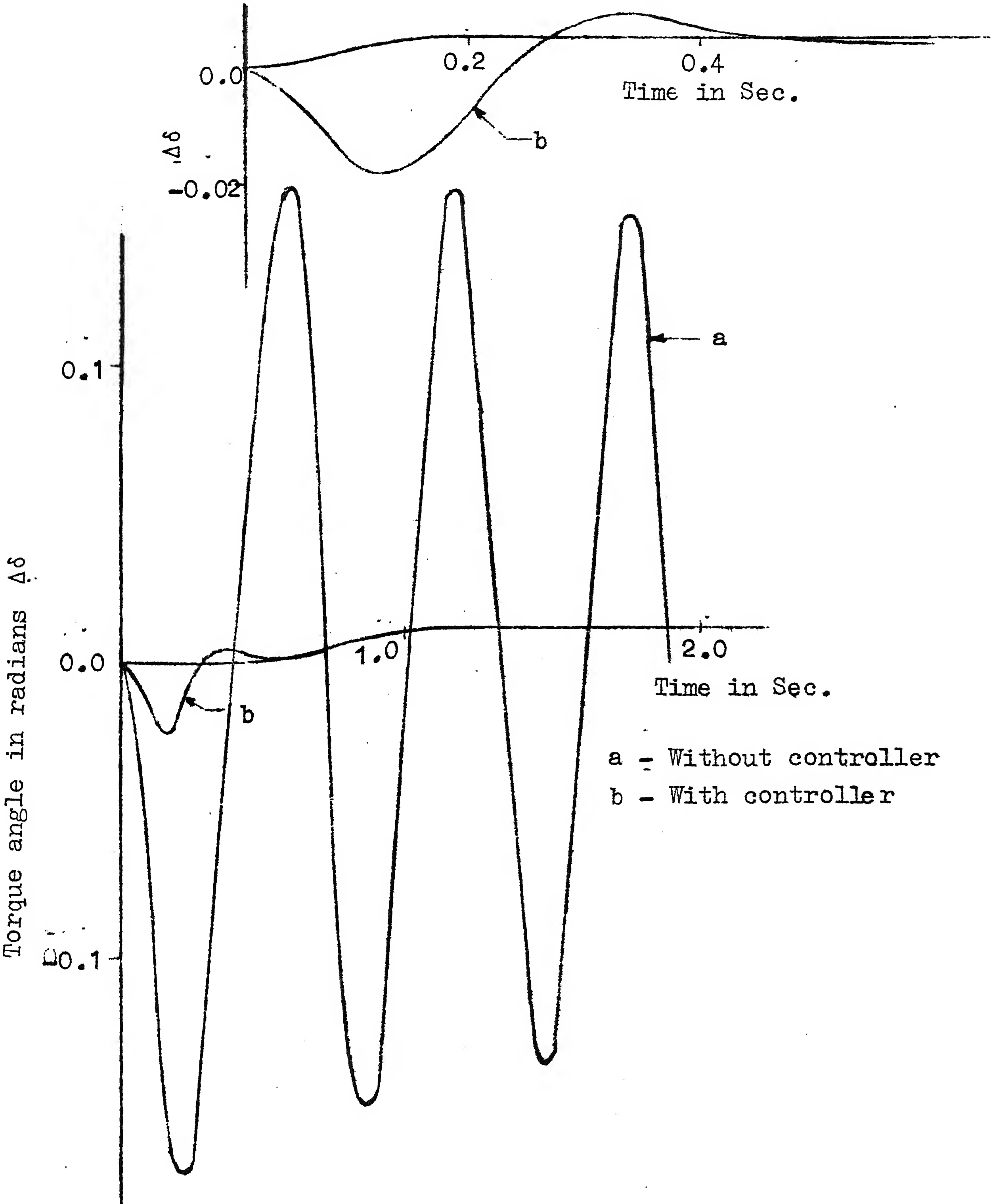

FIGURE 4.4 SYSTEM RESPONSE FOR $\Delta\delta$ FOR UNIT IMPULSE FUNCTION DISTURBANCE IN ΔE_{ref}

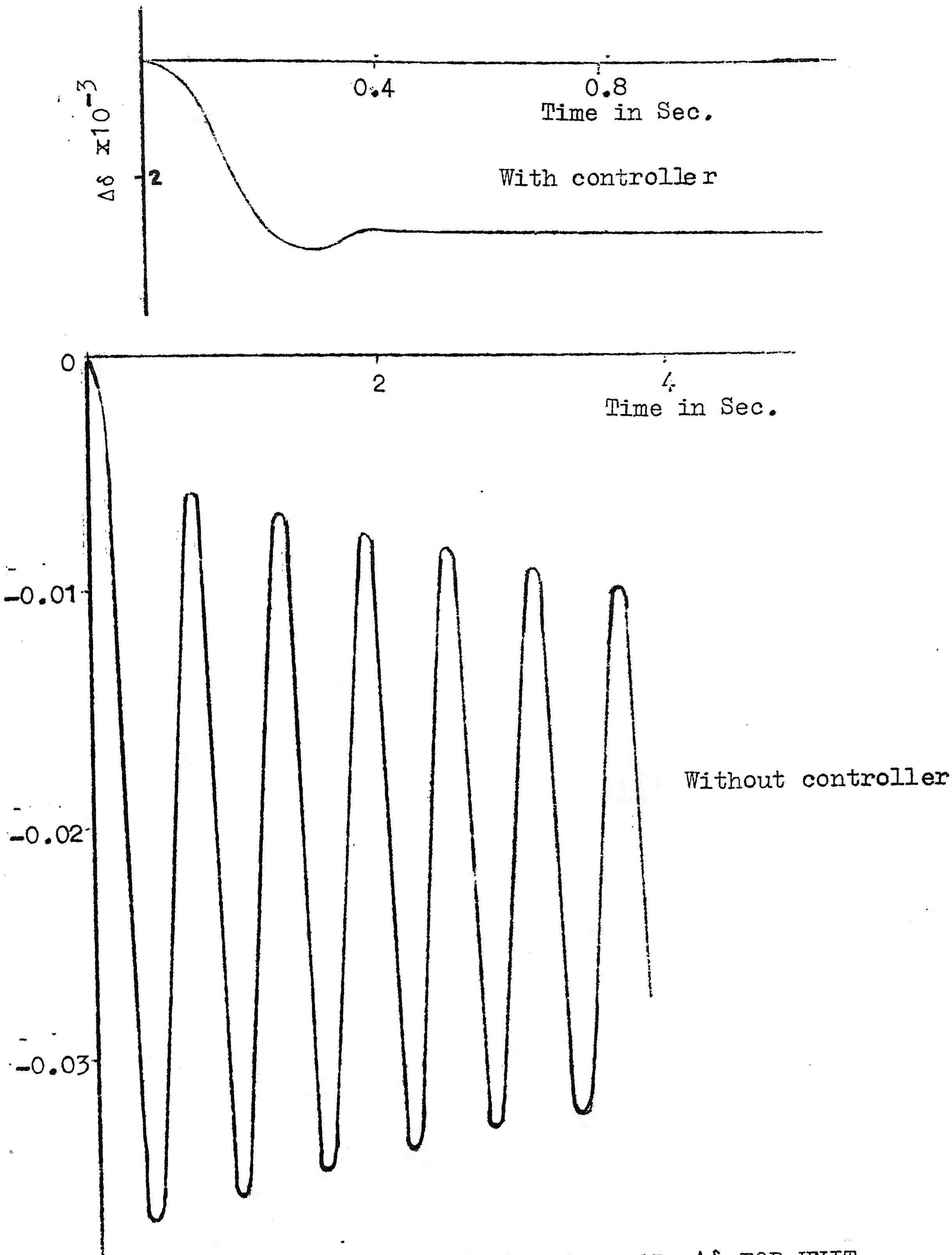

FIGURE 4.5 SYSTEM RESPONSE FOR $\Delta\delta$ FOR UNIT STEP FUNCTION DISTURBANCE IN ΔE_{ref}

Both the approaches result in increased feedback gains. To overcome the above difficulties, an alternate procedure of designing optimal modal controller which retains the computational simplicity of suboptimal design procedure (Section 4.6) and permits the division of the eigenvalues into large number of groups to yield better feedback gains is presented in this section. In this procedure reciprocal eigenvectors are to be computed only 'p' times and the optimization is done with respect to the following performance index

$$J_3 = \sum_{i=1}^{m} \sum_{j=1}^{n} (f_{ij})^{(q)2} \tag{4.12}$$

where

$$F^{(q)} = F^{(q-1)} + F_G = [f_{ij}^{(q)}]$$

= the resultant feedback matrix to shift eigenvalues of first q groups, (q=1,2,...,p)

F_G = the feedback matrix to shift the eigenvalues of the q^{th} group.

The feedback matrix F_G is obtained by minimizing the performance index J_3 (eqn.(4.12)). Thus it is to be noted that in this procedure the resultant feedback matrix obtained after shifting the eigenvalues of a group is to be optimized.

4.9.1 <u>Example</u> :

We illustrate the preceding procedure by means of an example. The power system example considered in this

section is the same as that in Section 4.4.1.

The performance index defined by eqn.(4.12) is to be minimized. The dominant eigenvalues are divided into the following three groups :

Group I : $-0.014 \pm j0.7986$

Group II : $-0.0772 \pm j0.1146$

Group III : -0.0572

By using the procedure of Section 4.9, the resultant optimal modal controller feedback matrix is obtained as

$$F = \begin{bmatrix} -0.056 & -0.915 & 1.73 & 1.180 & -1.769 & 0.036 & 0.344 & -1.725 \\ -0.854 & -4.979 & 0.13 & -0.058 & -0.077 & 8.405 & -1.317 & -2.646 \end{bmatrix}$$

The norm of the feedback matrix F as defined by eqn.(4.10) is 117.2. Thus this feedback matrix is better than the feedback matrices obtained for the suboptimal designs in Section 4.6.1 (Norms 233.23 and 236.07) and is comparable with the optimal modal controller matrix obtained in Section 4.4.1 (Norm: 96.2).

4.10 COMPARISON OF VARIOUS MODAL CONTROLLER DESIGNS

To compare the various modal controller matrices obtained for the ssystems considered, a performance index $J = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}^2$, where f_{ij} is the element of the modal controller feedback matrix F, is computed. Results for the power system (Section 2.2.1) problem and the load frequency

control (Section 2.3) problem are tabulated in Table 4.1 and 4.2 respectively.

From the Table 4.1 it is seen that considerable improvement in the performance index of the feedback matrix is obtained by eigenvalue grouping technique of Section 4.4 (col.(3), Table 4.1, and Col.(3), Table 4.2) as compared to the feedback matrix obtained without such grouping using method of Chapter 3 (cols.(1) and (2), Table 4.1 and cols. (1) and (2), Table 4.2). The suboptimal feedback matrix has a larger value of performance index (cols.(4) and (5), Table 4.1 and col.(4), Table 4.2) then the optimal feedback matrix (Col.(3), Table 4.1 and Col.(3), Table 4.2. The feedback matrix (Col.6, table 4.1) for the near optimal design (Section 4.8.1) is better then the suboptimal design (cols.(4) and (5), Table 4.1). The design procedure of Section 4.9 for large systems gives a better feedback matrix (col.(7), Table 4.1) then the suboptimal (Section 4.6.1) and near optimal (Section 4.8.1) feedback matrices (cols.(4), (5) and (6), Table 4.1) and the performance deterioration is negligible in comparison to the optimal design (col.(3), Table 4.1).

4.11 OPTIMAL MODAL CONTROL OF A LARGE SYSTEM

The numerical examples of power systems considered so far, in this chapter, have systems of order less than

ten and with two control inputs. The efficacy of the methods developed for the design of modal controllers for large systems can be demonstrated effectively by considering larger dynamic system. In this section a large system representing a chemical plant and described by 41 differential equations with 8 control inputs [73] is selected to demonstrate the techniques.

4.11.1 Description of the Chemical Plant

The following is a brief description of the chemical plant given in reference [73] based on the modal of Williams and Otto [78]. The plant consists of: (1) A stirred tank reactor, (2) A reaction cooler heat exchanger, (3) A decanter, (4) A distillation column unit, which includes a reboiler and condenser. A brief description and data for the linearized model of the chemical plant considered are given in Appendix C. The eigenvalues of the system matrix are:

-0.3306, -0.605, -0.815, $-0.99 \pm j0.04$, -4.905, -6.204, -6.773, -7.1, -9.792, -15.72, -16.2, $-18.87 \pm j10.48$, $-18.9 \pm j10.78$, -20.58, -20.6, $-21.6 \pm j9.987$, -35.19, -35.82, -41.2, -45.12, -45.5, -46.2, -58.59, -82.22, -82.6, -82.68, -82.71, -83.14, -176.9, -195.43, -350.0 (multiplicity 6), -610.25.

It is desired to improve the system response by assigning the first three dominant eigenvalues [-0.3306, -0.605, -0.815] to the desired closed loop locations

[-1.5, -2.0, -2.5] by modal control feedback.

The performance index defined in eqn.(4.12) is to be minimized. The modal controllers were designed to assign the dominant eigenvalues by applying the procedures outlined earlier in Sections 3.6, 4.7 and 4.10. The three designs of modal controller are presented below.

DESIGN I: Optimal Modal Controller without Eigenvalue Grouping

Applying the technique of Section 3.6, the following results are obtained:

$$\underline{\alpha} = [0.30059295\text{x}10^{7}\ -0.31186745\text{x}10^{+2}\ -0.42868258\text{x}10^{4}\ -0.42868258\text{x}10^{4}\ -0.96045876\text{x}10^{4}\ 0.11957965\text{x}10^{5}\ 0.14080716\ \ 0.52114441\text{x}10^{4}]^{T}$$

$$\underline{g} = [-0.11025\ \ 0.10370\ \ -0.075348\ \ -0.073791\ \ 0.10020\ \ 0.11455\text{x}10^{-6}\ \ 0.31581\text{x}10^{-7}\ \ 0.96441\text{x}10^{-9}\ \ 0.35765\text{x}10^{-9}\ \ 0.001838\ \ 0.0086633\ \ -0.0060509\ \ -0.0041796\ \ 0.0051829\ \ -0.21172\text{x}10^{-9}\ \ 0.046673\ \ 0.22110\ \ -0.15428\ \ -0.10653\ \ 0.13214\ \ 0.52745\text{x}10^{-7}\ \ 1.3318\ \ 4.1223\ \ -1.3359\ \ -0.44315\ \ 0.53715\ \ 0.033162\ \ 0.18197\ \ -0.12381\ \ -0.08457\ \ 0.10573\ \ -0.017673\ \ 0.11271\ \ -0.079455\ \ -0.061940\ \ 0.079465\ \ -0.042068\ \ 0.089896\ \ -0.064625\ \ -0.055312\ \ 0.072419\ ]^{T}$$

The modal control feedback matrix is

$$F = \underline{\alpha}\ \underline{g}^{T}$$

and the value of the performance index as defined in eqn.(4.12) is

$$J = 0.192 \times 10^{15}$$

DESIGN II: Optimal Modal Controller with Eigenvalue Grouping

Using the criterion of grouping the eigenvalues (Section 4.2.2), the eigenvalues are divided into the following two groups:

Group I : [-0.6056]

Group II : [-0.3306, -0.8156]

Applying algorithm given in Section 4.3 and optimizing the performance index defined in eqn. (4.12) the following results are obtained:

$$\underline{\alpha}^{(1)T} = [1.0 \ 1.0 \ 1.0 \ 1.0 \ 1.0 \ 1.0 \ -1.0 \ 1.0] \times 10^{7}$$

$$\begin{aligned}\underline{g}^{(1)} = [&-0.64982\times10^{-3} \ \ 0.66480\times10^{-4} \ \ -0.21082\times10^{-4} \\ &0.13315\times10^{-8} \ \ 0.21237\times10^{-4} \ \ 0.14268\times10^{-10} \\ &-0.41333\times10^{-10} \ \ -0.15117\times10^{-12} \ \ -0.32758\times10^{-11} \\ &-0.68275\times10^{-4} \ \ -0.59891\times10^{-5} \ \ -0.13521\times10^{-5} \\ &0.58303\times10^{-10} \ \ 0.99588\times10^{-6} \ \ -0.47097\times10^{-12} \\ &-0.17412\times10^{-2} \ \ -0.15274\times10^{-3} \ \ -0.34483\times10^{-4} \\ &0.16911\times10^{-8} \ \ 0.25398\times10^{-4} \ \ -0.55154\times10^{-10} \\ &-0.21799\times10^{-1} \ \ -0.42013\times10^{-2} \ \ -0.27945\times10^{-3} \\ &0.67191\times10^{-8} \ \ 0.10001\times10^{-3} \ \ -0.14058\times10^{-2} \\ &-0.12331\times10^{-03} \ \ -0.27839\times10^{-4} \ \ 0.13482\times10^{-8}\end{aligned}$$

$$0.20505\times10^{-4} \quad -0.83610\times10^{-3} \quad -0.34459\times10^{-4}$$
$$-0.19144\times10^{-4} \quad 0.10323\times10^{-8} \quad 0.15898\times10^{-4}$$
$$0.63594\times10^{-3} \quad 0.23987\times10^{-5} \quad -0.16464\times10^{-4}$$
$$0.95468\times10^{-9} \quad 0.14816\times10^{-4} \quad]^T$$

$$\underline{\alpha}^{(2)^T} = [-2.6024261 \; -0.14585953 \; -0.14609265 \; 0.14609265 \; -0.14606650 \; -0.14603529 \; 0.14610482 \; 1.9908551]\times10^7$$

$$\underline{g}^{(2)^T} = [0.21301\times10^{-2} \; -0.18491\times10^{-2} \; 0.92030\times10^{-3} \; 0.55276\times10^{-3}$$
$$-0.99732\times10^{-3} \quad 0.60359\times10^{-12} \quad 0.10212\times10^{-12}$$
$$0.36305\times10^{-14} \quad 0.33510\times10^{-14} \quad 0.34532\times10^{-4}$$
$$-0.13731\times10^{-3} \quad 0.60353\times10^{-4} \quad 0.31309\times10^{-4}$$
$$-0.48267\times10^{-4} \quad 0.73117\times10^{-16} \quad 0.88365\times10^{-3}$$
$$-0.35043\times10^{-2} \quad 0.15392\times10^{-2} \quad 0.79801\times10^{-3}$$
$$-0.12309\times10^{-2} \quad 0.13021\times10^{-12} \quad 0.67708\times10^{-3}$$
$$-0.63246\times10^{-1} \quad 0.12670\times10^{-1} \quad 0.33197\times10^{-2}$$
$$-0.49107\times10^{-2} \quad 0.77986\times10^{-3} \quad -0.28868\times10^{-2}$$
$$0.12440\times10^{-2} \quad 0.63351\times10^{-3} \quad -0.99102\times10^{-3}$$
$$0.98729\times10^{-3} \quad -0.18517\times10^{-2} \quad 0.84547\times10^{-3}$$
$$0.46399\times10^{-3} \quad -0.75967\times10^{-3} \quad 0.11533\times10^{-2}$$
$$-0.15217\times10^{-2} \quad 0.72205\times10^{-3} \quad 0.41434\times10^{-3}$$
$$-0.70274\times10^{-3}]$$

The modal controller feedback matrix is

$$F = \underline{\alpha}^{(1)} (\underline{g}^{(1)})^T + \underline{\alpha}^{(2)} (\underline{g}^{(2)})^T$$

and the value of the performance index as defined in eqn.(4.12) is ?

$$J = 0.49\times10^{-13}$$

DESIGN III: Near Optimal Modal Controller With Eigenvalue Grouping and Dominant Control Inputs

The eigenvalues are divided into two groups as given in Section 4.7 to identify dominant inputs, it is found that for Group I the dominant input is u_1 while for Group II u_1 and u_8 are the dominant inputs. In obtaining optimal feedback matrix the performance index defined in eqn.(4.12) is minimized. The following results are obtained:

$$\underline{\alpha}^{(1)T} = [1.0\text{x}10^{6}\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0]$$

$$\begin{aligned}\underline{g}^{(1)T} = [&-0.71913\text{x}10^{-3}\ 0.73570\text{x}10^{-4}\ -0.23331\text{x}10^{-4}\\
&0.14735\text{x}10^{-8}\ 0.23501\text{x}10^{-4}\ 0.15790\text{x}10^{-10}\\
&-0.45741\text{x}10^{-10}\ -0.16730\text{x}10^{-12}\ -0.36252\text{x}10^{-11}\\
&-0.75557\text{x}10^{-4}\ -0.66279\text{x}10^{-5}\ -0.14963\text{x}10^{-5}\\
&0.64521\text{x}10^{-10}\ 0.11021\text{x}10^{-5}\ -0.52119\text{x}10^{-12}\\
&-0.19269\text{x}10^{-2}\ -0.16903\text{x}10^{-3}\ -0.38161\text{x}10^{-4}\\
&0.18714\text{x}10^{-8}\ 0.28107\text{x}10^{-4}\ -0.61036\text{x}10^{-10}\\
&-0.24124\text{x}10^{-1}\ -0.46493\text{x}10^{-2}\ -0.30925\text{x}10^{-3}\\
&0.74357\text{x}10^{-8}\ 0.11068\text{x}10^{-3}\ -0.15557\text{x}10^{-2}\\
&-0.13647\text{x}10^{-3}\ -0.30808\text{x}10^{-4}\ 0.14920\text{x}10^{-8}\\
&0.22692\text{x}10^{-4}\ -0.92527\text{x}10^{-3}\ -0.38134\text{x}10^{-4}\\
&-0.21186\text{x}10^{-4}\ 0.11423\text{x}10^{-8}\ 0.17594\text{x}10^{-4}\\
&-0.70376\text{x}10^{-3}\ 0.26545\text{x}10^{-5}\ -0.18220\text{x}10^{-4}\\
&0.10565\text{x}10^{-8}\ 0.16396\text{x}10^{-4}]\end{aligned}$$

$$\underline{\alpha}^{(2)T} = [0.21930436\text{x}10^{7}\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ 0.0\ -0.862293\text{x}10^{7}]$$

$$\underline{g}^{(2)^T} = [\ -0.94256\times10^{-2}\ 0.89251\times10^{-2}\ -0.46599\times10^{-2}$$
$$-0.30583\times10^{-2}\ 0.52095\times10^{-2}\ -0.54128\times10^{-11}$$
$$-0.78789\times10^{-11}\ 0.14796\times10^{-12}\ -0.81820\times10^{-13}$$
$$-0.43713\times10^{-4}\ 0.68575\times10^{-3}\ -0.31505\times10^{-3}$$
$$-0.17322\times10^{-3}\ 0.25500\times10^{-3}\ 0.15664\times10^{-14}$$
$$-0.11296\times10^{-2}\ 0.17501\times10^{-1}\ -0.80349\times10^{-2}$$
$$-0.44151\times10^{-2}\ 0.65026\times10^{-2}\ 0.40679\times10^{-11}$$
$$0.37896\times10^{-1}\ 0.31894\ -0.66702\times10^{-1}$$
$$-0.18367\times10^{-1}\ 0.26029\times10^{-1}\ -0.12427\times10^{-2}$$
$$0.14413\times10^{-1}\ -0.64863\times10^{-2}\ -0.35050\times10^{-2}$$
$$0.52299\times10^{-2}\ -0.33595\times10^{-2}\ 0.91501\times10^{-2}$$
$$-0.43682\times10^{-2}\ -0.25671\times10^{-2}\ 0.39953\times10^{-2}$$
$$-0.45684\times10^{-2}\ 0.74546\times10^{-2}\ -0.37029\times10^{-2}$$
$$-0.22924\times10^{-2}\ 0.36864\times10^{-2}\ ]$$

The modal controller feedback matrix is

$$F = \underline{\alpha}^{(1)}(\underline{g}^{(1)})^T + \underline{\alpha}^{(2)}\ (\underline{g}^{(2)})^T$$

and the value of the performance index as defined in eqn.(4.12) is

$$J = 0.86\times10^{13}$$

With either of the above modal controllers i.e. Design I, Design II and Design III, the locations of the first six dominant eigenvalues of the closed loop system are:

$$-0.99 \pm j10.4,\ -1.5,\ -2.0,\ -2.5,\ -4.905$$

Davison and Chadha [73] obtained the incomplete feedback law

$$u_8 = 1.0\text{x}10^7\ [0.51\ x_{22} - 0.85\ x_{23}]$$

to have the locations of the first six dominant eigenvalues of the closed loop system as

$$-0.8156,\ -1.0 \pm 0.0368,\ -1.6 \pm j7.169,\ -2.1826.$$

The value of the performance index as defined by eqn.(4.10) for this feedback matrix is

$$J = 0.94\text{x}10^{14}$$

4.11.2 Discussion of Results

The performance indices for the Designs II and III are comparable. In Design III, for shifting eigenvalues in group I only a unique feedback matrix exists, since u_1 is the only input. While for the second group u_1 and u_8 qualify as the dominant inputs. Therefore the optimization is to be done only with respect to $\alpha_1^{(2)}$ and $\alpha_8^{(2)}$ elements of $\underline{\alpha}^{(2)}$. This will reduce the computational effort. Furthermore, in Design III, feedback is to be provided to inputs u_1 and u_8. Therefore this design is easily implementable. The performance index values for the Design II and III are better than Design I where eigenvalues are shifted by applying the algorithm of Section 3.4. Considering the locations of the first six dominant eigenvalues of the closed loop system, the performance index J for Design II and Design III is better than that of Davison and Chadha [73].

In reference [73], the incomplete state feedback law attempts to shift only one eigenvalue but there is a significant disturbance in the locations of several other eigenvalues.

4.12 CONCLUSIONS

In this chapter simple and computationally efficient algorithms have been presented to obtain optimal modal controllers. Grouping of dominant eigenvalues has been shown to yield improved results, which could be significant in many cases. A criterion for grouping of dominant eigenvalues has been given. A basis for identifying dominant control inputs to shift eigenvalues of each group is suggested. Arranging feedback to only the dominant inputs simplifies the control configuration and also reduces computational effort. Sub-optimal and near-optimal modal control design procedures have been suggested for designing controllers for large systems. Keeping in view the need for obtaining procedures to determine optimal modal controllers with reduced computational effort, a new performance index has been suggested. Numerical examples of modal controller designs for power systems have been considered in detail and the computational results obtained by applying the procedures developed have been discussed. To demonstrate the suitability and effectiveness of the procedures suggested in this chapter, a numerical example of the

design of controllers for a large system (forty first order system with eight control inputs) representing a chemical plant has been also solved. The techniques used to design modal contollers for the above problem can be applied to power systems which are more complex than the one considered here.

CHAPTER 5

WIDE RANGE MODAL CONTROLLERS FOR POWER SYSTEMS

5.1 INTRODUCTION

The controller schemes discussed in earlier chapters were designed for the linear models obtained by linearizing the exact nonlinear equations describing the power system around an operating point. The operating conditions, however, change with the load demand on the system. The controllers designed for a particular operating condition may be inappropriate for other operating conditions. There is a need, therefore, to design controllers which give satisfactory system performance over a range of operating conditions.

Moussa and Yu [26] considered the above problem and suggested incorporation of correcting feedback, in addition to the conventional feedback controller designed for a particular operating point, to compensate for small variation in the operating conditions. Application of optimal linear regulator theory incorporating the trajectory sensitivity aspect has also been proposed by Elmetwally and Rao [32,33]. Habibullah and Yu [24] have suggested an iterative algorithm based on application of optimal linear regulator theory and eigenvalue search technique.

In this chapter a scheme to determine feedback, in addition to the modal controller designed for a particular

operating point has been developed. This additional feedback obtained from eigenvalue sensitivity analysis reduces the variations in the closed loop system poles with change in operating conditions. This feedback is a function of the system state variables and those parameter of the system that are subjected to change. This technique has the advantage and simplicity of the modal control approach; furthermore it enables realization of the desired system performance over a range of operating conditions. It has been applied to a power system problem and the computational results have been given. The problem of implementation of this scheme has also been discussed.

The proposed controller may be called an Adaptive Modal Controller or a Wide Range Modal Controller.

5.2 DESIGN OF WIDE RANGE MODAL CONTROLLER

5.2.1 Conventional Modal Controller

Consider the linear, time-invariant, n^{th} order system with m inputs

$$\dot{\underline{x}} = A_o \underline{x} + B_o \underline{u} \tag{5.1}$$

where the suffix 'o' denotes the value at the nominal operating point

It has been shown in Section 4.2 , that the dynamic response of the nominal system can be improved by the modal control feedback law,

$$\underline{u} = F_o \underline{x} \, , \tag{5.2}$$

$$F_o = \sum_{q=1}^{p} \underline{\alpha}^{(q)} (\underline{g}_o^{(q)})^T \tag{5.3}$$

which shifts the 'ℓ' dominent eignevalues of A_o, by state feedback to the desired closed loop locations without disturbing the locations of the remaining eigenvalues. It is assumed that the dominant eigenvalues are divided into 'p' groups.

5.2.2 Wide Range Modal Controller

Consider now the perturbed system

$$\dot{\underline{x}} = (A_o + \Delta A)\, \underline{x} + (B_o + \Delta B)\, \underline{u} \tag{5.4}$$

where A and B are variations in the system and control matrices due to the perturbations in system operating conditions.

The feedback law

$$\underline{u} = \underline{u}_o = F_o\, \underline{x} \tag{5.5}$$

is such that for the unperturbed system, the 'ℓ' dominant eigenvalues of the system are assigned to the closed loop locations, $\rho_1, \rho_2, ---, \rho_\ell$, without disturbing the locations of the non-dominant eigenvalues. However, perturbations in system parameters will, in general, result in perturbations of the above closed loop eigenvalues. The objective in the design of a wide range modal controller is to provide additional feedback such that the first order variations of the above eigenvalues with respect to system parameter variations is rendered zero. For this purpose the eigenvalue sensitivity reduction technique given by Tzafestas and

Paraskevopoulas [79] for single input systems is modified to make it applicable for multi-input systems.

The dominant eigenvalues are divided into p groups for the eigenvalue assignment problem. Now, consider a feedback, in addition to the given in eqn. (5.5), of the form

$$\Delta \underline{u} = \left(\sum_{q=1}^{p} \underline{\alpha}^{(q)} \right) \Delta \underline{\theta}^{T} H \underline{x} \tag{5.6}$$

where $\underline{\theta}$ is the vector of those elements of system matrix A_o and the matrices $B_o \underline{\alpha}^{(1)}$, $B_o \underline{\alpha}^{(2)}$, ---, $B_o \underline{\alpha}^{(p)}$ which undergo perturbation, and $\Delta\underline{\theta}$ is the resulting perturbation of the $\underline{\theta}$ vector. Let $\underline{\theta}$ be of dimension r. Then H is an r x n matrix and the i^{th} row of H matrix gives the correction required to reduce the variation in closed loop pole locations due to perturbation in the i^{th} element of $\underline{\theta}$. The elements of H matrix are derived as follows.

Let the eigenvalues of the closed loop disturbed system be denoted as $\lambda_i^c(\underline{\theta})$, (i=1,2,...,n). Expanding $\lambda_i^c(\underline{\theta})$ in a Taylor series expansion around nominal value $[\lambda_i^c(\underline{\theta})]_o$

$$\lambda_i^c(\underline{\theta}) = [\lambda_i^c(\underline{\theta})]_o + \left[\frac{\partial \lambda_i^c(\underline{\theta})}{\partial \theta_1}, \frac{\partial \lambda_i^c(\underline{\theta})}{\partial \theta_2}, \ldots, \frac{\partial \lambda_i^c(\underline{\theta})}{\partial \theta_r} \right] \Delta \underline{\theta} + \text{high order terms} \tag{5.7}$$

when the correction given by eqn. (5.6) is added to the control law obtained from eqn. (5.~~3~~2), the resultant closed loop system matrix becomes

$$A_{CL} = A + B \left[\sum_{q=1}^{p} \underline{\alpha}^{(q)} \{ (\underline{g}_o^{(q)})^T + \Delta \underline{\theta}^T H \} \right]$$

Let $\underline{b}^{(q)} \triangleq B \, \underline{\alpha}^{(q)}$

Then

$$A_{CL} = A + \sum_{q=1}^{p} \underline{b}^{(q)} \left[(\underline{g}_o^{(q)})^T + \Delta \underline{\theta}^T H \right] \ldots \tag{5.8}$$

It has been shown in Appendix D, that the eigenvalue sensitivity, [80], for the above system is given by

$$\left(\frac{\partial \lambda_i^c}{\partial \theta_j}\right)_o = [T_r \, R^c(\lambda_i^d)]_o^{-1} \, [R^c(\lambda_i^d)]_o \circledast [(A_{\theta_j})_o + \sum_{q=1}^{p} (\underline{b}_{\theta_j}^{(q)})_o \, (\underline{g}_o^{(q)})^T + \underline{b}_o \, \underline{h}_j^T + \text{H.O.T.}], \; (i=1,2,\ldots,n), \; (j = 1,2,\ldots,r) \tag{5.9}$$

where

$$\underline{b}_o \triangleq \sum_{q=1}^{p} \underline{b}_o^{(q)}$$

$$\underline{b}_o^{(q)} \triangleq B_o \, \underline{\alpha}^{(q)} ;$$

A_{θ_j} and $\underline{b}_{\theta_j}^{(q)}$ are the partial derivatives of A and $\underline{b}^{(q)}$ with respect to element θ_j of $\underline{\theta}$; $R^c(\lambda_i^d) = \text{Adjoint}\,[I\lambda_i^d - A_o - \sum_{q=1}^{p} \underline{b}_o^{(q)} (\underline{g}_o^{(q)})^T]$; $\underline{h}_j^T$ is the j^{th} row of H and the suffix 'o' denotes that the expressions are evaluated at the nominal operating conditions. λ_i^d , (i = 1,2,...n) denotes the desired closed loop pole locations corresponding to $\rho_1, \rho_2, \ldots, \rho_\ell$, $\lambda_{\ell+1}, \ldots, \lambda_n$ respectively and the operator $\circledast$ is as defined in Appendix D.

The second term of eqn. (5.7) is zero if $\left(\frac{\partial \lambda_i^c}{\partial \theta_j}\right)_o = 0$, $(j = 1,2,\ldots,r)$ and $(i=1,2,\ldots,n)$. Then, from eqn. (5.9)

$$[R^c(\lambda_i^d)]_o \circledast [\underline{b}_o \underline{h}_j^T] = - [R^c(\lambda_i^d)]_o \circledast [(A_{\theta_j})_o + \sum_{q=1}^{p} (\underline{b}_{\theta_j}^{(q)})_o (\underline{g}_o^{(q)})^T]$$

or

$$[[R^c(\lambda_i^d)]_o \underline{b}_o]^T \underline{h}_j = - [R^c(\lambda_i^d)]_o \circledast [(A_{\theta_j})_o + \sum_{q=1}^{p} (\underline{b}_{\theta_j}^{(q)})_o (\underline{g}_o^{(q)})^T] \qquad (5.10)$$

or

$$Q \underline{h}_j = - \underline{e}_j \quad , \quad (j=1,2,\ldots,n) \qquad (5.11)$$

where

$$Q = \begin{bmatrix} [[R^c(\lambda_1^d)]_o \underline{b}_o]^T \\ \vdots \\ [[R^c(\lambda_n^d)]_o \underline{b}_o]^T \end{bmatrix} \quad \text{--- n x n matrix}$$

$$\underline{e}_j = \begin{bmatrix} [R^c(\lambda_1^d)]_o \circledast [(A_{\theta_j})_o + \sum_{q=1}^{p} (\underline{b}_{\theta_j}^{(q)})_o (\underline{g}_o^{(q)})^T] \\ \vdots \\ [R^c(\lambda_n^d)]_o \circledast [(A_{\theta_j})_o + \sum_{q=1}^{p} (\underline{b}_{\theta_j}^{(q)})_o (\underline{g}_o^{(q)})^T] \end{bmatrix}$$

--- n-vector

The solution for $\underline{h}_j$, $(j=1,2,\ldots,r)$ is obtained by solving the equations (5.11). If Q is singular, the equations in the set (5.11) corresponding to the rows of Q which are linearly independent are retained and the remaining equations are discarded. A solution for $\underline{h}_j$ can then be obtained. However the solution obtained is not unique.

5.3 WIDE RANGE MODAL CONTROLLER FOR A POWER SYSTEM

5.3.1 System Model

The power system considered is the same as that in Chapter 3 Section 3.6.1. The numerical data for this system is given in Section 2.2.1. The desired closed loop pole locations for the nominal system are the same as those given in Sections 3.6.1 and 4.4.1.

It is desired to design a wide range modal controller so that the deviation in the closed loop eigenvalue locations for a wide range of operating conditions is small. The operating conditions are given by the real power P and the reactive power Q delivered by the generator. The elements of the system matrix A, which depend on P and Q are a_{21}, a_{23}, a_{31} and a_{41} (see Section 2.2.1). The elements of the control matrix B are independent of the operating conditions.

5.3.2 Results

Two modal controller designs are considered for shifting the five dominant eigenvalues of the uncontrolled system to the required closed loop locations at the nominal operating conditions. In design I all the five dominant eigenvalues are considered to be in one group and the pole shifting procedure given in Section 5.2.1 is applied. In design II, dominant eigenvalues are divided into two groups. The first three dominant eigenvalues are in group 1 and the next two are in group 2. The pole assignment procedure is then applied to the two groups as indicated in Section 5.2.1. Using sensitivity considerations and the procedure of Section 5.2.2 additional feedback controller is designed for each of the above cases. Closed loop system pole locations for the above two designs are computed for various operating conditions. The results are tabulated in Tables 5.1 to 5.5. The operating conditions (real and reactive power delivered by the machine) considered also include the cases considered by Habibullah and Yu, [24]. In table 5.5 closed loop system eigenvalues for the controllers with and without additional feedback for certain operating conditions are compared.

<u>DESIGN I</u>

Modal Controller Design [see Section 3.6.1]

$\underline{\alpha} = [-25.1027 \; , \; 94.6869]^T$

$\underline{g}_o = [0.055,\ 0.104,\ -0.165,\ 0.267,\ -2.328,\ 8.3466,\ -0.65,\ 0.935]^T$

$\underline{b}_o = [0.0,\ 0.0,\ 0.0,\ 0.0,\ -25.1027,\ 0.0,\ 94.6869,\ 0.0]^T$

Design of Additional Feedback

$$\Delta\underline{\theta} = [\Delta a_{21},\ \Delta a_{23},\ \Delta a_{31},\ \Delta a_{41}]^T$$

$$H = \begin{bmatrix} 0.104 & 0.640 & -0.314 & 0.201 & -2.170 & 10.130 & -0.575 & 2.216 \\ 0.164 & 0.026 & 0.677 & 0.593 & -3.438 & 10.661 & -0.917 & 0.760 \\ -0.097 & -0.315 & 0.460 & 0.201 & 0.328 & 1.291 & 0.087 & 1.045 \\ -0.412 & 0.201 & 0.201 & -0.532 & 4.992 & -5.888 & 1.323 & 3.611 \end{bmatrix}$$

The wide range modal controller feedback matrix is

$$F_{wr} = \underline{b}_o\,\underline{g}_o^{\,T} + \underline{b}_o\,\Delta\underline{\theta}^T H \qquad (5.12)$$

DESIGN II

Modal Controller Design [see Section 4.8.1]

$\underline{\alpha}_o^{(1)} = [0.0 \quad 1.0]^T$

$\underline{g}_o^{(1)} = [0.881,\ -5.028,\ 0.086,\ -0.1,\ 0.183,\ 8.38,\ -1.32,\ -2.678]^T$

$\underline{b}_o^{(1)} = [0.0,\ 0.0,\ 0.0,\ 0.0,\ 3.122,\ 0.0,\ 3.296,\ 0.0]^T$

$\underline{\alpha}^{(2)} = [1.0 \quad 0.0]^T$

$\underline{g}_o^{(2)} = [0.481,\ 0.184,\ 1.818,\ 1.204,\ -1.766,\ -6.566,\ 0.093,\ -2.774]^T$

$\underline{b}_o^{(2)} = [0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.3749,\ 0.0,\ 3.22,\ 0.0]^T$

$\underline{\bar{b}}_o = [0.0,\ 0.0,\ 0.0,\ 0.0,\ 1.0,\ 0.0,\ 1.0,\ 0.0\ ]^T$

Design of Additional Feedback

$$\Delta\underline{\theta} = [\Delta a_{21},\ \Delta a_{23},\ \Delta a_{31},\ \Delta a_{41}]^T$$

$$H = \begin{bmatrix} -9.039 & -5.137 & -1.474 & -0.339 & 0.296 & 29.194 & -0.296 & 12.670 \\ 0.135 & -1.319 & -0.546 & -0.138 & 0.120 & -6.650 & -0.120 & -0.396 \\ -1.019 & -1.474 & -2.691 & -0.526 & 0.400 & 14.257 & -0.400 & 4.981 \\ -0.269 & -0.339 & -0.526 & -0.122 & -0.099 & 3.174 & -0.099 & 1.060 \end{bmatrix}$$

The wide range modal controller feedback matrix is

$$F_{wr} = \sum_{i=1}^{2} \underline{b}_o^{(i)} \underline{g}_o^{(i)T} + \underline{b}_o \Delta \underline{\theta}^T H \tag{5.13}$$

5.3.3 Discussion of Results

The results given in Tables 5.1 to 5.5 show that the variations in the closed loop pole locations have been rendered small over a fairly wide range of system operating condition by the Wide Range Modal Controller scheme.

Examination of the two designs indicates an interesting aspect of the additional feedback in its relation to the nominal modal control feedback. The correction required to the nominal control could be viewed as being indicated by the magnitudes of the elements of the matrix H. Larger the magnitudes of the elements of the matrix H, larger the correction provided by the additional feedback. It is observed that the correction provided by additional feedback is more in Design II than that of Design I. This can be explained as follows: The nominal modal control feedback gains in Design I are larger than those in Design II. Therefore the relative variations in the elements of the

TABLE 5.1 : CLOSED LOOP SYSTEM EIGENVALUES FOR DIFFERENT REACTIVE LOADINGS

S.No.	Q (p.u)	Closed loop system eigenvalues with Wide Range Modal Controller.
1	0.7	-13.71, -1.553, -0.7122 ± j1.272, -0.1814 ± j0.4802, -0.2858, -0.1559
2	0.6	-13.71, -1.4, -0.7028 ± j1.176, -0.2637 ± j0.5145, -0.2834, -0.1655
3	0.5	-13.71, -0.6875 ± j1.072. -1.222, -0.3646 ± j0.5589, -0.2809, -0.1743
4	0.4	-13.71, -0.6554 ± j0.9384, -1.001, -0.5046 ± j0.6443, -0.2783, -0.1820
5	0.3	-13.71, -0.7979 ± j0.7151, -0.7713, -0.4751 ± j0.8715, -0.2761, -0.1883
6	0.2	-13.71, -0.9092 ± j0.7171, -0.6508, -0.4226 ± j0.9025, -0.2745, -0.1927
7	0.1	-13.71, -0.9481 ± j0.7245, -0.6072, -0.4048 ± j0.9137, -0.2736, -0.1950.
8 *	0.034	-13.71, -0.9539 ± j0.7255, -0.6006, -0.4021 ± j0.9155, -0.2736, -0.1954
9	0.0	-13.71, -0.9523 ± j0.7251, -0.6027, -0.4028± j0.9151, -0.2735, -0.1953
10	-0.1	-13.71, -0.9332 ± j0.7212, -0.6244, -0.4114 ± j0.9094, -0.2739, -0.1940
11	-0.2	-13.71, -0.8924 ± j0.7167, -0.6733, -0.4290 ± j0.8971 ± 0.2739, -0.1915.
12	-0.3	-13.71, -0.8263 ± 0.7184, -0.6944, -0.4575 ± j0.8768, -0.2737, -0.1884
13	-0.4	-13.71, -0.726 ± j0.7478, -0.8584, -0.5068 ± j0.8379, -0.2731, -0.1848
14	-0.5	-13.71, -0.6622 ± j0.8922, -0.9746, -0.6622 ± j0.8922, -0.2721, -0.1808

* Nominal operating condition P_0=0.735, Q_0=0.034, v_t=1.05

TABLE 5.2 : CLOSED LOOP SYSTEM EIGENVALUES FOR DIFFERENT REACTIVE LOADINGS DESIGN II

S.No.	Q (p.u.)	Closed loop system eigenvalues with Wide Range Modal Controller
1	0.7	-13.72, -0.9737 ± j0.7154, -0.6231, -0.3966 ± j0.9389, -0.3013, -0.1175 .
2	0.5	-13.71, -0.9672 ± j0.7154, -0.6033, -0.3988 ± j0.9275, -0.2902, -0.1587 .
3	0.4	-13.71, -0.9649 ± j0.7159, -0.5979, -0.3995 ± j0.9235, -0.2846, -0.1739 .
4	0.3	-13.71, -0.9630 ± j0.7167, -0.5953, -0.3999 ± j0.9204, -0.2797, -0.1851 .
5	0.2	-13.70, -0.9616 ± j0.7177, -0.5954, -0.4001 ± j0.9181, -0.2761, -0.1922 .
6	0.1	-13.70, -0.9605 ± j0.7191, -0.5978, -0.4000 ± j0.9163, -0.2743, -0.1951 .
7*	0.034	-13.70, -0.9600 ± j0.7201, -0.6004, -0.3999 ± j0.9154, -0.2740, -0.1951 .
8	0.0	-13.70, -0.9597 ± j0.7207, -0.6021, -0.3998 ± j0.9150, -0.2741, -0.1945 .
9	-0.1	-13.70, -0.9592 ± j0.7225, -0.6080, -0.3993 ± j0.8140, -0.2748, -0.1714 .
10	-0.2	-13.69, -0.9588 ± j0.7226, -0.6154, -0.3982 ± j0.9132, -0.276, -0.1866 .
11	-0.3	-13.69, -0.9581 ± j0.7209, -0.6239, -0.3979 ± j0.9125, -0.2771, -0.1810 .
12	-0.4	-13.68, -0.9586 + j0.7239, -0.6335, -0.3970 ±j0.9121, -0.2777, -0.1748 .
13	-0.5	-13.68, -0.9589 ± j0.7320, -0.6442, -0.3960 ± j0.9116, -0.2778, -0.1683 .
14	-0.7	-13.67, -0.9604 ± j0.7376, -0.6691, -0.3937± j0.9114, -0.2758, -0.1541 .

* Normal operating condition P_0=0.735, Q_0=0.034, v_t=1.05.

TABLE 5.3 : CLOSED LOOP SYSTEM EIGENVALUES FOR DIFFERENT OPERATING CONDITIONS, DESIGN I

S.No.	P p.u.	Q p.u.	Closed loop system eigenvalues with Wide Range Modal controller
1*	1.25	0.45	-13.71, -1.518, -0.7051 ± j1.258, -0.3168, -0.1811 ± j0.4649, -0.1746 .
2*	1.20	0.34	-13.71, -1.2141, -0.6787 ± j1.103, -0.3405 ± j0.5036, -0.3150, -0.1876 .
3*	1.15	0.25	-13.71, -0.8307, -0.6283 ± j0.5992, -0.5945 ± j0.9259, -0.3078, -0.1975 .
4*	0.952	0.015	-13.71, -0.9974 ± j0.7323, -0.5357, -0.3875 ± j0.9312, -0.2731, -0.2028 .
5*	0.7	-0.15	-13.71, -0.9514 ± j0.7245, -0.6026, -0.4035 ± j0.9149, -0.2739, -0.1953 .
6*	0.5	-0.225	-13.71, -0.9658 ± j0.7192, -0.5631, -0.4037 ± j0.9226, -0.2804, -0.1991 .
7*	0.3	-0.256	-13.71, -1.055 ± j0.7414, -0.4153, -0.3696 ± j0.9539, -0.3081, -0.2097 .
8	0.3	0.0256	-13.71, -1.050 ± j0.7436, -0.4618, -0.3691 ± j0.9518, -0.2710, -0.2115 .
9	0.952	-0.15	-13.71, -1.001 ± j0.7336, -0.5332, -0.3855 ± j0.9324, -0.2717, -0.2032 .
10	1.15	-0.25	-13.71, -1.112 ± j0.7712, -0.4064, -0.3404 ± j0.9755, -0.2355 ± j0.0419.
11	1.20	-0.34	-13.71, -1.143 ± j0.7852, -0.3845, -0.3262 ± j0.9891, -0.2290 ± j0.06 .
12	1.25	-0.45	-13.71, -1.176 ± j0.8014, -0.3710, -0.3117 ± j1.004, -0.2174 ± j0.0754 .
13	1.4	-0.6	-13.71, -1.307 ± j0.8786, -0.3339, -0.24 ± j1.063, -0.1771 ± j0.1121 .
14	1.5	-0.65	-13.71, -1.382 ± j0.9298, -0.3237, -0.1935 ± j1.11, -0.1528 ± j0.1252 .

* operating conditions considered by Habibullah and Yu [24]

TABLE 5.4 : CLOSED LOOP SYSTEM EIGENVALUES FOR DIFFERENT OPERATING CONDITIONS DESIGN II

S.No.	P p.u.	Q p.u.	Closed loop system eigenvalues with Wide Range Modal Controller
1	1.5	0.65	-13.67, -0.9782 ± j0.7073, -0.7053, -0.4069 ± j0.9078, -0.3439, -0.0526 .
2*	1.25	0.45	-13.68, -0.9720 ± j0.7049, -0.6474, -0.4025 ± j0.9046, -0.3165, -0.1332 .
3*	1.20	0.34	-13.68, -0.9702 ± j0.7051, -0.637, -0.4017 ± j0.9031, -0.3048, -0.1562,
4*	1.15	0.25	-13.68, -0.9692 ± j0.7062, -0.6318, -0.4009 ± j0.9033, -0.2942, -0.1712 .
5 *	0.95	0.015	-13.68, -0.9653 ± j0.7144, -0.6246, -0.3989 ± j0.9086, -0.2739, -0.1900 .
6 *	0.7	-0.15	-13.7, -0.9588 ± j0.7217, -0.5986, -0.4 ± j0.9159, -0.2742, -0.1952 .
7 *	0.5	-0.225	-13.71, -0.9522 ± j0.7279, -0.5804, -0.4018 ± j0.9198, -0.2769, -0.1975 .
8 *	0.3	-0.256	-13.73, -0.9491 ± j0.7269, -0.5479, -0.4053 ± j0.9289, -0.2675, -0.2131 .
9	0.7	0.15	-13.70, -0.959 ± j0.7211, -0.5971 , -0.4001, ± j0.9163, -0.2739, -0.1958.
10	0.95	_0.15	-13.68, -0.965 ± j0.7148, -0.6266, -0.3987 ± j0.9085, -0.2732, -0.1896 .
11	1.25	-0.45	-13.65, -0.9806 ± j0.7031, -0.6934, -0.3961 ± J0.09069, -0.2568, -0.1671 .
12	1.4	-0.6	-13.63, -0.988± j0.6907, -0.7141, -0.3974 ± j09055, -0.2488, -0.1648 .
13	1.5	-0.65	-13.63, -0.9895 ± j0.6818 -0.718, -0.3997 ± j0.9005, -0.2476, -0.1707.

* operating conditions considered by Habibullah and Yu [24]

TABLE 5.5: COMPARISON OF CLOSED LOOP SYSTEM POLES

S.No.	P. p.u.	Q p.u.	Closed loop system eigen-values with conventional modal controller	Closed loop system eigenvalues with Wide Range modal controller	Design
1	2	3	4	5	6
1	1.5	0.65	-13.7, -0.7912, ± j0.756, -0.96 ± j0.72, -0.27, 0.0007 ± j0.145.**	-13.67, -0.9782 ± j0.7073, -0.7053, -0.4069 ± j10.978, -0.3439, -0.053.	II
2*	1.25	0.45	-13.7, -0.963 ± j0.724, -0.622 ± 0.760, -0.273, -0.1719 ± j0.196.	-13.68, -0.972 ± j0.704, -0.647, -0.4 ± j0.95, -0.316, -0.1332.	II
3*	1.2	0.34	-13.7, -0.96 ± j0.72, -0.55 ± j0.78, -0.275, -0.243 ± j0.159.	-13.68, -0.97 ± j0.705, -0.637, -0.4 ± j0.90, -0.3048, -0.1562.	II
4*	1.15	0.25	-13.7, -0.96 ± 0.72, -0.498 ± j0.81, -0.29 ± j0.08, -0.2884.	-13.68, -0.964 ± j0.706, -0.631, -0.4 ± j0.9, -0.2942, -0.171.	II
5	0.735	0.8	-13.7, -0.965 ± j0.728 -0.68 ± j0.75, -0.2757, -0.11 ± j0.217.	-13.72, -0.9779 ± j0.7159, -0.6372, -0.3952, ± j0.946, -0.306, -0.1.	II
6	1.5	-0.65	-13.71, -1.63, -0.818 ± j0.527, -0.719 ± j1.33, -0.164, -0.081.	-13.71, -1.38 ± j0.929, -0.3237, -0.1935 ± j1.1, -0.153 ± j0.12.	II
7*	1.25	0.45	-13.71, -1.793, -0.8151 ± j1.387, -0.3157, -0.1369 -0.0505 ± j0.4206.**	-13.71, -1.518, -0.7051 ± j1.258, -0.3168, -0.1811 ± j0.4649, -0.1746.	I

1	2	3	4	5	6
8	1.2	0.34	-13.71, -1.71, -0.8028 ± j1.034, -0.3165, -0.1437, -0.003 ± j0.4588.	-13.71, -1.241, -0.6787 ± j1.103, -0.3405 ± j0.503, -0.315, -0.1876.	I
9*	1.15	0.25	-13.71, -1.627, -0.7872 ± j1.292, -0.317, -0.1498, -0.057 ± j0.4884.	-13.71, -0.8307, -0.6283 ± j0.5992, -0.5945 ± j0.926, -0.3078, -0.1975.	I
10	0.735	0.6	-13.71, -1.52, -0.8162 ± j1.217, -0.288, -0.457, -0.0964 ± j0.471.	-13.71, -1.4, -0.7028 ± j1.17, -0.2637 ± j0.5154, - 0.2834, -0.1655.	I
11	0.735	0.8	-13.71, -1.661, -0.8413 ± j0.1293, -0.2903, -0.1311, -0.008 ± j0.4024.	-13.71, -1.692, -0.7185 ± j1.364, -0.288, -0.1457, -0.1099 ± j0.4445.	I

* Operating conditions considered by Habibullah and Yu [24]

** Unstable system

closed loop system matrix due to the change in operating conditions are smaller in Design I. Hence the amount of correction required in Design I is smaller.

5.4 COMMENT ON PHYSICAL REALIZATION

In the implementation of the wide range modal controller, state feedback proportional to the change in system parameters is to be provided. This can be achieved in the case of the power system considered in Section 5.3 using advanced techniques in adaptive control such as model reference adaptive control schemes which can be implemented on line. For the problem of power system considered, the real and reactive power delivered by the machine are easily measurable quantities.

5.5 CONCLUSIONS

A simple and effective method of designing wide range modal controllers for the power system is presented in this chapter. The controller so designed reduces the variations in the closed loop system eigenvalues to variations in operating conditions over a fairly wide range. This assures satisfactory system performance over a wide range of system operating conditions. It is observed that if the feedback gains in the modal controller design for nominal operating condition are reduced significantly by optimization, then large correction to the nominal control is required when the

operating condition changes. These observations are in agreement with the result in classical control theory, which states that larger the negative feedback, the more insensitive is the system performance to variations in the system parameters [81].

CHAPTER 6

ACCESSIBLE STATE FEEDBACK MODAL CONTROLLERS

6.1 INTRODUCTION

The modal controllers designed in earlier chapters require feedback of all the state variables of the system. This may be difficult to achieve in practice. If the system is observable, it is possible to reconstruct the state variables to the required degree of accuracy from the system outputs with the help of 'observers' [82]. The reconstructed state variables can then be fed back to the conventional feedback controllers. However, the disadvantage of this technique, especially in high order systems, is the additional dynamics due to the observers. Davison [83] obtained results regarding assignment of $\ell(\leq n)$ open loop poles to desired closed loop locations by linear output feedback. In this procedure, the unassigned poles may get disturbed. This may cause the closed loop response to be unsatisfactory. Fallside and Seraji [84, 85] proposed a method for the output feedback pole assignment problem with unity rank matrices. In this case also the locations of unassigned poles may be disturbed.

In this chapter the design of modal controllers utilizing accessible state or output feedback to assign the dominant poles to desired locations, without disturbing the

locations of the remaining poles of the open loop system, has been considered. The possibility of optimizing feedback gains has been investigated. Numerical examples have been given to illustrate the procedures developed. The chapter also includes design of a dynamical modal controller scheme. The schemes developed have been applied to a power system problem.

6.2 MODAL CONTROLLERS USING ACCESSIBLE STATE FEEDBACK

Consider the linear, n^{th} order system with m control inputs described by

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{6.1}$$

Let

$$\underline{x} = [\underline{x}_1^T \; , \; \underline{x}_2^T]^T$$

where

$\underline{x}_1$ = the r-vector of inaccessible states.

It is required to find a feedback law $\underline{u} = F_2\,\underline{x}_2$ to shift the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_\ell$ to preassigned locations $\rho_1, \rho_2, \ldots, \rho_\ell$ respectively without disturbing the locations of the remaining eigenvalues of the uncontrolled system. It is assumed that the modes λ_1 to λ_ℓ are controllable.

Let F_2 be constrained to be of the dyadic form:

$$F_2 = \underline{\alpha}\;\underline{g}_2^T \tag{6.2}$$

where $\underline{\alpha}$ is an m-vector and the feedback gain vector $\underline{g}_2$ is of dimension (n-r).

The gain vector $\underline{g}$ of dimension n for the complete state feedback pole assignment problem is given by eqn.(3.6).

$$\underline{g} = V_L \underline{c} \tag{6.3}$$

where

$$c_i = i^{th} \text{ component of the } \ell\text{-vector } \underline{c}$$

$$= [k_i/(\underline{v}_i^T B \underline{\alpha})] \tag{6.4}$$

and
$$k_j = [\prod_{i=1}^{\ell} (\rho_i - \lambda_j)]/[\prod_{\substack{i=1 \\ i \neq j}}^{\ell} (\lambda_i - \lambda_j)], \quad (j=1,2,\ldots,\ell) \tag{6.5}$$

It is to be noted that for the pole assignment problem,

$$c_i \neq 0, \ (i=1,2,\ldots,\ell)$$

Let

$$V_L = [\ V_1^T \ , \ V_2^T\ ]^T$$

$$\underline{g}^T = [\ \underline{g}_1^T \ , \ \underline{g}_2^T\ ]$$

and

$$\underline{c}^T = [\underline{c}_1^T \ , \ \underline{c}_2^T\ ]$$

where V_1, $\underline{g}_1$ and $\underline{c}_1$ are of dimension $r \times \ell$, r and s respectively, s being the rank of V_1. If the state vector $\underline{x}_1$ is not to be fedback, then $\underline{g}_1 = \underline{0}$. Therefore, from eqn. (6.3)

$$V_1 \underline{c} = \underline{0} \tag{6.6}$$

A necessary and sufficient condition for non-trivial solution $\underline{c}$ in eqn. (6.6) is $s \leq \ell - 1$. This implies that $r \leq \ell - 1$, that is, the number r of inaccessible states should be less than the number of eigenvalues to be shifted. Since rank $[V_1] = s$, $(\ell - s)$ elements of $\underline{c}$ can be arbitrarily selected

and the rest of the elements determined from eqns. (6.6). It is required that none of c_i's determined should be zero. In many problems, solution for $\underline{c}$ satisfying the above conditions can be obtained.

Equations (6.3) are rewritten below for obtaining $\underline{\alpha}$:

$$V_L^T \; B \, \underline{\alpha} = \underline{z} \tag{6.7}$$

where the i^{th} element of the ℓ-dimensional $\underline{z}$ vector is $z_i = (k_i/c_i)$. Let rank $[V_L^T \; B]$ = rank $[V_L^T \; B \; \vdots \; \underline{z}]$. Then a solution for $\underline{\alpha}$ can be obtained. Then F_2 is obtained after determining $\underline{g}$ from eqns.(6.2).

It is to be noted that if $s < r$ and/or $\ell < m$, the freedom available in the choice of $\underline{c}$ and/or $\underline{\alpha}$ can be utilized to obtain optimal feedback gains. Furthermore, if it is found that the solution of eqns. (6.7) does not exist (this is more likely to happen if ℓ is large), then the eigenvalues to be moved may be divided into groups and the procedure may be applied repetitively to the groups.

6.3 EXAMPLES

Three numerical examples are considered in this section. The first two are simple problems to demonstrate the suitability of the procedure developed in Section 6.2, for designing accessible state feedback modal controllers. In example 3, accessible feedback modal controller design

for a power system is given.

6.3.1 Example 1 : Accessible Feedback Modal Controller for a Simple System

The system considered has the following system and control matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -6 & -5 & 1 \\ 0 & 0 & -1 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (6.8)$$

The measurable states are

$$\underline{x}_2 = [\, x_1 \, , x_2 \,]^T$$

The eigenvalues of uncontrolled system are -2,-3 and -1.
A modal controller which utilizes feedback from the measurable states $\underline{x}_2$ is to be designed to shift the open loop eigenvalues -2 and -3 to locations -4 and -5.

Solution:

$$r = 1 \; ; \; \ell = 2 \; ; \; m = 2 \; ; \; n = 3 \; ;$$

$$k_1 = -6 \; ; \; k_2 = 2 \; ; \; c_1 = 0.5 \text{ for } c_2 = 1.0$$

$$\underline{\alpha} = [2.5, 1.0]^T \; ; \; \underline{g} = [-4 \, , -4 \, , 0]^T$$

and the modal controller feedback matrix is

$$F_2 = \begin{bmatrix} -10 & -10 \\ -4 & -4 \end{bmatrix}$$

6.3.2 Example 2 : Optimal Modal Controller with Accessible State Feedback

Consider the example 1, in which the B matrix is changed to

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution:

Since $\ell < m$, the elements of $\underline{\alpha}$ vector are selected to minimize the performance index

$$J = \sum_{i=1}^{3} \alpha_i^2 + \sum_{j=1}^{3} g_j^2$$

Then,

$$\underline{\alpha} = [-1.915, -3.566, 1.0]^T$$

$$\underline{g} = [2.56, 2.56, 0.0]^T$$

and the modal controller matrix with measurable state feedback is

$$F_2 = \begin{bmatrix} -4.9 & -4.9 \\ -9.13 & -9.13 \\ 2.56 & 2.56 \end{bmatrix}$$

6.3.3 Example 3: Accessible State Feedback Modal Controller for a Power System

The power system considered is the same as that given in Sections 2.3.1 and 3.6.1.

The complete state vector is

$$\underline{x} = [\Delta\delta, n, \Delta\Psi_f, \Delta v_f, \Delta v_s, \Delta g, \Delta g_f, \Delta h]^T$$

The state variable $\Delta\Psi_f$ represents the flux linkages of the field winding. This quantity cannot be directly measured. A modal controller which does not require feedback of $\Delta\Psi_f$ is to be designed to shift the dominant eigenvalues $[-0.0114 \pm j0.7986, -0.0572, -0.0772 \pm j0.1146]$ to the closed loop locations $[-0.4 \pm j0.915, -0.6, -0.96 \pm j0.72]$. Since $r = 1$, we take $\ell = 2$. The number of dominant modes is five. In order to have 3 groups with two eigenvalues in each group, one of the non-dominant eigenvalues is also shifted, but only very slightly. Thus the eigenvalues to be shifted are divided into following three groups:

Group I : $[-0.0114 \pm j0.7986]$

Group II : $[-0.0772 \pm j0.1146]$ and

Group III : $[-0.0572, -0.1953]$

and the respective desired closed loop locations are, $[-0.4 \pm j0.915]$, $[-0.96 \pm 0.72]$ and $[-0.6, -0.196]$.

The procedure of Section 6.2 is applied in a repetitive manner. The resultant modal control law is found to be

$$F = \begin{bmatrix} -12.3452 & -461.1443 & 0.0 & -0.1196 & -10.6961 & -3801.6890 & -124.4492 & -1295.2454 \\ -0.6185 & -20.9747 & 0.0 & -0.0960 & -0.6996 & -222.0193 & 7.6098 & -73.5784 \end{bmatrix} \tag{6.9}$$

Note that the column corresponding to $\Delta\Psi_f$ in the above modal controller feedback matrix has zero entries.

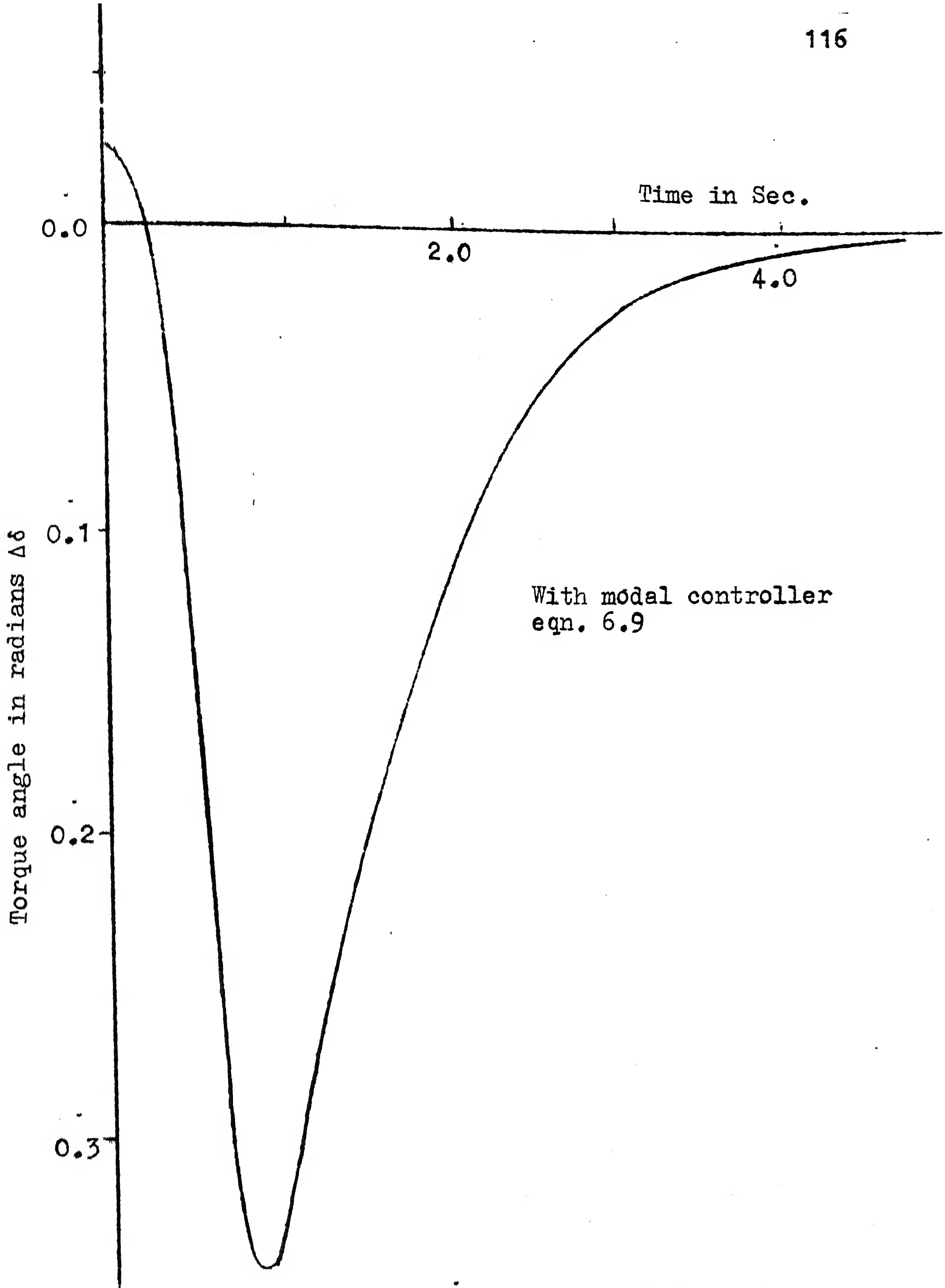

FIGURE 6.1 SYSTEM RESPONSE FOR $\Delta\delta$

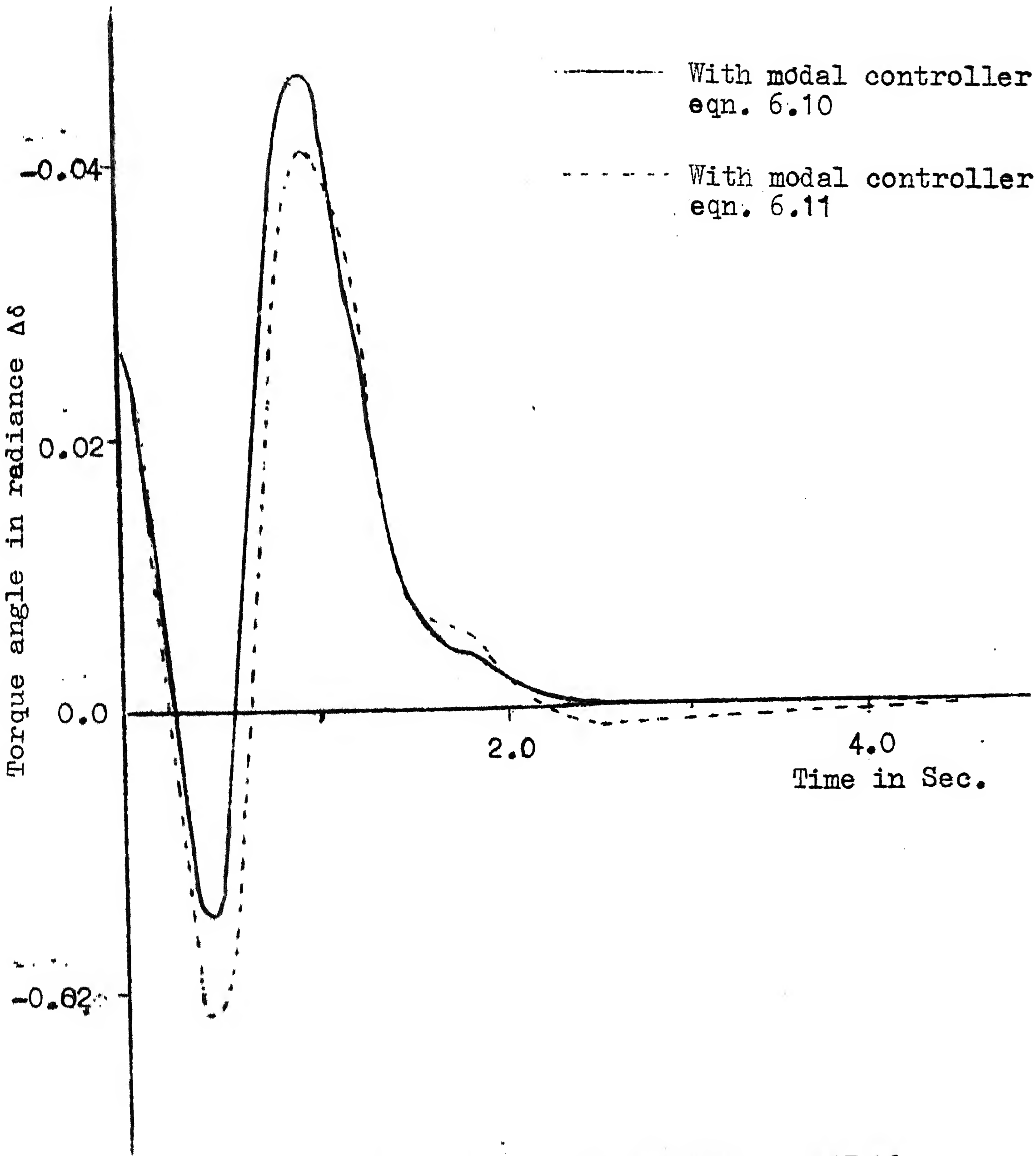

FIGURE 6.2 SYSTEM RESPONSE FOR Δδ

It may be possible to have a better feedback matrix in term of the performance index given by eqn. (3.8), if instead of the non-dominant eigenvalue -0.1953, some other eigenvalue from the non-dominant set is shifted. For example, when the eigenvalue -13.4 is shifted to -12.33 alongwith the dominant eigenvalues, the feedback matrix is

$$F = \begin{bmatrix} 110.5713 & 1282.31 & 0.0 & 16.7521 & -9.7971 & -312.3934 & -131.6718 & 218.8719 \\ 0.9469 & 13.99 & 0.0 & 0.2001 & -0.1447 & -4.6833 & 8.0858 & 2.7420 \end{bmatrix} \tag{6.10}$$

The feedback gains can be further reduced if eigen values at location -13.4 is shifted towards the right half plane without appreciably affecting the closed loop performance. When it is assigned -5.28, the feedback matrix is

$$F = \begin{bmatrix} 46.9245 & 528.2866 & 0.0 & 6.7510 & -4.060 & -145.8445 & 43.6062 & 85.8360 \\ 0.4722 & 8.3518 & 0.0 & 0.1255 & -0.102 & -3.4411 & 9.3930 & 1.7499 \end{bmatrix} \tag{6.11}$$

The closed loop system responses with the above controllers and the initial conditions considered in Section 3.6.1 are given in Figure 6.3.

6.4 OUTPUT FEEDBACK MODAL CONTROLLER

Consider the system eqn. (6.1). The p outputs of the system are given by the output equation

$$\underline{y} = D\,\underline{x} \tag{6.12}$$

Let rank [D] = p. It is required to find a feedback law

$$\underline{u} = F_o \underline{y} \tag{6.13}$$

to shift $\lambda_1, \lambda_2, \ldots, \lambda_\ell$ to preassigned locations $\rho_1, \rho_2, \ldots, \rho_\ell$ without disturbing the locations of the remaining eigenvalues. It is assumed that the modes λ_1 to λ_ℓ are output controllable. Let F_o be constrained to be of dyadic form

$$F_o = \underline{\alpha}\ \underline{\beta}^T \tag{6.14}$$

where the gain vector $\underline{\beta}$ is of dimension p.

Now, consider the case when it is possible to generate $\underline{g}$ of eqn. (6.3), in the row space of D, that is,

$$\underline{g} = D^T \underline{\beta} \tag{6.15}$$

Then

$$\underline{u} = \underline{\alpha}\ (D^T\ \underline{\beta})^T\ \underline{x} \tag{6.16}$$

The control law (6.16) utilizes only output feedback. From (6.3) and (6.15) we have

$$D^T\ \underline{\beta} = V_L\ \underline{c} \tag{6.17}$$

or

$$\overline{D} \begin{bmatrix} \underline{\beta} \\ \cdots \\ \underline{c} \end{bmatrix} = \underline{0} \tag{6.18}$$

where $\overline{D} = [D^T \ \vdots\ - V_L]$ is an nx(p+ℓ) matrix. Let rank $[\overline{D}]$ = r. Then a necessary and sufficient condition for non-trivial solution for $[\underline{\beta}^T \ \vdots\ \underline{c}^T]$ in eqn. (6.18) is

$$p + \ell > r \tag{6.19}$$

It is required that none of the c_i's determined should be zero. In many problems, solution for $\underline{c}$ satisfying the above criterion ~~be~~ is (may be) found. Equations (6.7) can now be solved to obtain $\underline{\alpha}$. The output feedback modal controller matrix F_o can then be obtained from eqn. (6.13).

If rank $[V_L^T B] < m$, then the freedom available in the choice of $\underline{\alpha}$ may be utilized to set some of the elements of $\underline{\alpha}$ to be zero. This implies that feedback is arranged to only some of the available inputs, resulting in a simpler control configuration.

6.4.1 Optimal Modal Controller

If $p+\ell-r > 1$ in eqn.(6.19) and/or rank $[V_L^T B] < m$, then freedom available in the choice of $\underline{c}$ and/or $\underline{\alpha}$ in eqn. (6.18) and eqn.(6.7) can be utilized to obtain optimal feedback gains. The following performance index may be selected for feedback gain minimization:

$$J = \sum_{i=1}^{m} \sum_{j=1}^{p} \alpha_i^2 \beta_j^2 \qquad (6.20)$$

6.5 ALGORITHMS

6.5.1 Algorithm To Design Output Feedback Modal Controller

1) Determine the reciprocal eigenvectors of the system matrix A, corresponding to 'ℓ' eigenvalues ($\lambda_1, \lambda_2, \ldots, \lambda_\ell$) to be assigned. Also check that modes λ_1 to λ_ℓ are output controllable.

ii) Calculate k_j, $(j=1,2,\ldots,\ell)$ from eqn.(6.7).

iii) Compute the rank of the matrix $\bar{D}$ (eqn.(6.18)). If condition eqn.(6.19) is satisfied, then go to step (iv), otherwise stop.

iv) Obtain the solutions for the vector $\underline{\beta}$ and $\underline{c}$ from eqns. (6.18) such that $c_i \neq 0$, $(i=1,2,\ldots,\ell)$.

v) Compute $[k_i/c_i]$, $(i=1,2,\ldots,\ell)$.

vi) Find the values for α_i, $(i=1,2,\ldots,m)$. by solving eqns. (6.7).

vii) Obtain the modal controller matrix F from eqn.(6.14).

viii) Stop.

6.5.2 Algorithm to Design Output Feedback Optimal Modal Controller

i) Determine the reciprocal eigenvectors of the system matrix A, corresponding to 'ℓ' eigenvalues $(\lambda_1, \lambda_2, \ldots, \lambda_\ell)$ to be assigned. Also check that λ_1 to λ_ℓ are output controllable.

ii) Compute r the rank of matrix $[\bar{D}]$ eqn. (6.18). If $p+\ell-r$ eqn.(6.19) is not greater zero, then stop. Let $p+\ell-r = r_1$.

iii) Calculate k_j, $(j=1,2,\ldots,\ell)$ from eqn. (6.15).

iv) Set iteration count n=1.

v) Set objective function value $J^{(n)} = 0$, where the objective function J is of the form (6.20) and superscript n denotes the iteration count.

vi) Choose suitable values for r_1 elements of the $\underline{c}$ and let these elements constitute the vector $\underline{c}_2$. Obtain the solution for $\underline{\beta}$ and the remaining element of $\underline{c}$ from eqns. (6.18). The vector $\underline{c}_2$ should be such that $c_i \neq 0$, $(i=1,2,\ldots,\ell)$. If the above requirement cannot be met then stop.

vii) Compute $z_i = [k_i/c_i]$, $(i=1,2,\ldots,\ell)$.

viii) Find r_2, the rank of matrix $[V_L^T\, B]$ in eqns. (6.7). If rank $[V_L^T\, B] \neq$ rank $[V_L^T\, B : \underline{z}]$, modify $\underline{c}_2$ in step (vi), to make it equal. If this is not possible then stop.

ix) Choose suitable values for $(m-r_2)$ elements of $\underline{\alpha}$. Let these elements form a vector $\underline{\alpha}_2$. Solve eqns.(6.17) for the remaining elements of $\underline{\alpha}$.

x) Calculate the new objective function value $J^{(n+1)}$ from eqn. (6.20). If $|J^{(n+1)} - J^{(n)}|$ is less than a specified small positive number, go to step (xvi).

xi) Change $\underline{c}_2$ and $\underline{\alpha}_2$ vectors as $\underline{c}_{2\,new} = \underline{c}_2 + s\,(\Delta\,\underline{c}_2)$ and $\underline{\alpha}_{2\,new} = \underline{\alpha}_2 + s\,(\Delta\,\underline{\alpha}_2)$ respectively, where s is the step length and $\Delta\,\underline{c}_2$ and $\Delta\underline{\alpha}_2$ are the negative gradients of the Lagrangian function with respect to $\underline{c}_2$ and $\underline{\alpha}_2$ respectively.

xii) Solve eqns. (6.18) for $\underline{\beta}$ and the remaining elements of $\underline{c}$. Check, whether $c_i \neq 0$, $(i=1,2,\ldots,\ell)$. Otherwise modify appropriately the step length s in step(xi).

xiii) Repeat step vii.

xiv) Solve eqns. (6.17) for $\underline{\alpha}$. If solutions for $\underline{\alpha}$ do not exist then change suitably the step length s in step(xi).

xv) Increment the iteration count n by one and go to step(x).

xvi) Obtain the modal controller feedback matrix F_o from eqn. (6.14).

xvii) Stop.

6.6 NUMERICAL EXAMPLES OF THE DESIGN OF OUTPUT FEEDBACK MODAL CONTROLLERS

Three numerical examples are considered in this section to demonstrate the suitability of the algorithms developed in the previous section for designing output feedback modal controllers. In example 2, the feedback gains are optimized.

6.6.1 Example 1 : Output Feedback Modal Controller for a Simple System

Consider the system (6.8) of example 6.3.1. Let the output matrix be

$$D = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0.5 \end{bmatrix}$$

The output feedback modal controller is to be designed for the desired closed loop, pole locations

mentioned in example 6.3.1.

Solution:

In this problem:

$p = 2$, $\ell = 2$, $m=2$, $r=2$ and $p+\ell-r = 2 > 0$. Taking $c_2 = 1.0$, we obtain from eqns. (6.18)

$$c_1 = 1.0$$
$$\underline{\beta}^T = [1,-1]$$

Then

$$\underline{\alpha}^T = [8 \quad 2]$$

and the feedback matrix

$$F_o = \begin{bmatrix} 8 & -8 \\ 2 & -2 \end{bmatrix}$$

6.6.2 Example 2 : Optimization Problem

Consider the example 6.6.1, except that B is changed to

$$B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Let the performance index to be minimized to be

$$J = \sum_{i=1}^{m} \sum_{j=1}^{p} \alpha_i^2 \beta_j^2$$

Solution:

$$\underline{\alpha}^T = [9.33 \quad 13.33 \quad -5.33]$$

and the modal controller feedback matrix is

$$F_o = \begin{bmatrix} -4.66 & -4.66 \\ -6.66 & -6.66 \\ 2.66 & 2.66 \end{bmatrix}$$

6.6.3 Example 3 : Problem Considered by Sridhar and Lindorff [87] and Sirisena and Choi [88]

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

It is desired to shift eigenvalues ($\lambda = 1, 2$) of the uncontrolled system to locations ($\rho = -1, -2$) without disturbing the eigenvalues ($\lambda = -3, -4$).

Solution:

In this case:

$p = 2$, $\ell = 2$, $m = 2$, $r = 2$, $p + \ell - r = 2 > 0$,

$k_1 = 6$ and $k_2 = -12$. Taking $c_2 = 1.0$, we obtain from eqns. (6.18)

$c_1 = 1.0$

and $\underline{\beta}^T = [\,1 \quad 0\,]$

Then $\underline{\alpha}^T = [\ 6 \quad -12\ ]$

and the feedback matrix is

$$F_o = \begin{bmatrix} 6 & 0 \\ -12 & 0 \end{bmatrix}$$

The closed loop system matrix then is

$$[A + B\,\underline{\alpha}\,\underline{\beta}^T D] = \begin{bmatrix} 7 & 6 & 0 & 0 \\ -12 & 10 & 0 & 0 \\ 6 & 6 & -3 & 0 \\ -6 & -6 & 0 & -4 \end{bmatrix}$$

If now an attempt is made to shift the eigenvalues ($\lambda = -3, -4$) to some other locations then it is found that the rank $[\bar{D}] = 4$ and $p + \ell - r \not> 0$, which violates eqns.(6.19). These modes, therefore, now cannot be shifted by this technique.

6.7 ACCESSIBLE STATE FEEDBACK DYNAMICAL MODAL CONTROLLER

An interesting scheme for designing feedback controllers for systems with inaccessible state variables has been given by Liou et al [89]. The procedure enhances the dimension of the state space by considering the control inputs also to be state variables. In the augmented system thus obtained, the control variables are the derivatives of the original control variables. An advantage of this approach is that the original control variables will not

have discontinuities while the system is in operation and the controller will be 'smooth acting'. This may imply longer controller life. Liou et al [89] use infinite time optimal linear regulator theory for the determination of the feedback controller for the augmented system. This approach, however, may not be computationally attractive for higher order system [Section 3.1]. The formulation of Liou et al is used here, but the solution to the problem is obtained by the application of modal control technique developed earlier.

6.7.1 Development:

The system (6.1) is augmented such that

$$\dot{\underline{z}} = \bar{A}\,\underline{z} + \bar{B}\,\underline{w} \tag{6.21}$$

where

$$\underline{z} = [\underline{x}^T \quad \underline{u}^T]^T ,$$

$$\bar{A} = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} ,$$

$$\bar{B} = [0^T , \quad I_m]^T$$

I_m = mxm identity matrix and

$\bar{A}$ and $\bar{B}$ are (n+m) x (n+m) and (n+m) x m system and control matrices respectively of the augmented system. A modal control law to achieve the desired closed loop poles may be obtained by the procedures given in Sections 3.4 and 4.4 as

$$\underline{w} = \bar{F}\,z \tag{6.22}$$

Eqn. (6.22) can be rewritten as

$$\underline{w} = \bar{F}\,\underline{x} + H\,\underline{u} \tag{6.23}$$

where

$\bar{F} = [\ F\ :\ H\ ]$,

F = (mxn) state feedback matrix and

H = (mxm) matrix associated with the feedback of $\underline{u}$

The equation (6.23) can also be expressed as

$$\dot{\underline{u}} - H\,\underline{u} = F_1\,\underline{x}_1 + F_2\,\underline{x}_2 \tag{6.24}$$

where

$F = [\ F_1\ :\ F_2\ ]$,

$\underline{x}_1$ = r-vector of inaccessible states and

$\underline{x}_2$ = (n-r) vector of accessible states

The system eqns. (6.1) can be written as

$$\begin{bmatrix} \dot{\underline{x}}_1 \\ \dot{\underline{x}}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \underline{x}_1 \\ \underline{x}_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \underline{u} \tag{6.25}$$

If the (n-r) x r matrix A_{21} has rank r, then $(A_{21}^T\ A_{21})^{-1}$ exists. Eqns. (6.25) then yields

$$\underline{x}_1 = a\,(\dot{\underline{x}}_2 - A_{22}\ \underline{x}_2 - B_2\,\underline{u}) \tag{6.26}$$

where

$$a = [\ A_{21}^T\ A_{21}\ ]^{-1}\ A_{21}^T$$

Then, the dynamic modal controller law obtained from eqn.(6.24) and (6.26) is

$$\dot{\underline{u}} - (\ H - F_1 a B_2\ )\,\underline{u} = F_1\,a\,\dot{\underline{x}}_2 - [\ F_1 a A_{22} - F_2\ ]\,\underline{x}_2 \tag{6.27}$$

This can be rewritten as

$$\dot{\underline{u}} + G_1 \underline{u} = G_2 \dot{\underline{x}}_2 + G_3 \underline{x}_2$$

where

$$G_1 = -[H - F_1 a B_2] , \tag{6.28}$$

$$G_2 = F_1 a \quad \text{and} \tag{6.29}$$

$$G_3 = - [F_1 a A_{22} - F_2] \tag{6.30}$$

6.8 EXAMPLE: DYNAMICAL MODAL CONTROLLER FOR A POWER SYSTEMπ

The power system considered in Section 6.3.3 is also considered here. The state variables $\Delta\Psi_f$ and $\Delta \dot{h}$ are considered to be inaccessible. To apply the b procedure of Section 6.7, the state variables are rearranged in the state vector such that :

$$\begin{aligned} \underline{x}_1 &= [\Delta\Psi_f \quad h]^T \\ \underline{x}_2 &= [n, \Delta v_f, \Delta v_s, \Delta g, \Delta g_f, \Delta \delta]^T \end{aligned} \tag{6.31}$$

The augmented system (6.21) has the following seven dominant eigenvalues

0.0, 0.0, $-0.014 \pm j0.7986$, -0.0572, $-0.0772 \pm j0.1146$.

A modal controller is to be designed to shift the above eigenvalues to the following locations

$-0.4 \pm j0.915$, -0.5, -0.6, $-0.96 \pm j0.72$, -1 .

The set of dominant eigenvalues contains a pair of repetitive eigenvalues at the origin. The modal control

procedure of Chapter 3 cannot therefore be directly applied. To overcome this difficulty, either of the following approaches may be used:

<u>Approach I</u>

Observing the system and the control matrices of the augmented system, it is found that the following control law shifts the two zero eigenvalues to the locations -1.0 and -0.5 respectively, without disturbing the rest of the eigenvalues:

$$\underline{w} = F_0 \, \hat{\underline{z}}$$

where

$$F_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix}$$

The closed loop system matrix A_1 with the above feedback is

$$A_1 = \bar{A} + \bar{B} \, F_0$$

Then a control law F_1 to shift the remaining dominant eigenvalues to the desired closed loop locations is found. The resultant modal control matrix F is obtained by summing the two feedbacks laws F_0 and F_1. It is found to be

$$F = \begin{bmatrix} 5.2732 & 1.2668 & -2.9498 & -10.7221 & 1.13 & -0.3545 & 1.3796 & 0.6008 & -2.8396 & 0.5837 \\ -0.3040 & 0.4029 & -0.8079 & -4.7426 & -1.15 & 4.0930 & 0.9751 & -8.0436 & -0.4792 & -1.7460 \end{bmatrix}$$

The dynamic modal control law utilizing feedback from accessible states is then given by

$$\dot{\underline{u}} + G_1 \underline{u} = G_2 \dot{\underline{x}}_2 + G_3 \underline{x}_2$$

where

$$G_1 = \begin{bmatrix} 2.8396 & -0.5837 \\ 0.4792 & 1.7460 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.4552 & -13.2406 & 0.0 & 0.0 & 0.0 & 0.0 \\ -6.0936 & -4.4660 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 5.2889 & -0.5472 & -21.0894 & -11.1235 & 1.1300 & 1.6776 \\ -0.5148 & -0.2089 & -6.9263 & 0.6320 & -1.1522 & 0.5116 \end{bmatrix}$$

Approach II

The dominant eigenvalues are divided into two groups, such that each group has only one zero eigenvalue:

Group I : [0.0, $-0.0114 \pm j0.7986$, -0.0572]

Group II : [0.0, $-0.0772 \pm j0.1146$]

A modal controller feedback matrix F_1 is obtained to shift the eigenvalues of group I to the desired locations. Then a feedback matrix F_2 is computed to shift the eigenvalues of group II to the assigned locations. The resultant feedback matrix F is obtained by adding the two matrices F_1 and F_2. The matrices F_1 and F_2 are selected to minimize the performance index

$$J = \sum_{i=1}^{m} \sum_{j=1}^{n} f_{ij}^2$$

where f_{ij} is the element of the feedback matrix F corresponding to i^{th} row and j^{th} column.

The result and modal control law for the augmented system is found to be

$$\underline{w} = \begin{bmatrix} 2.2407 & 0.8016 & -2.9995 & -1.5569 & 0.2332 & -0.0700 & 0.5978 & 0.5219 & -3.2861 & -0.0041 \\ -3.8417 & 0.1574 & -0.6008 & 2.7968 & -2.0503 & 4.3960 & 0.6940 & -8.9629 & 4.6945 & 1.2995 \end{bmatrix} \underline{z}$$

and the dynamic modal control law utilizing feedback from accessible states is

$$\dot{\underline{u}} + G_1 \underline{u} = G_2 \dot{\underline{x}}_2 + G_3 \underline{x}_2$$

where

$$G_1 = \begin{bmatrix} 3.2860 & 0.0041 \\ -4.6945 & 1.2995 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 0.4552 & -13.2406 & 0.0 & 0.0 & 0.0 & 0.0 \\ -6.0936 & -4.466 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 5.2889 & -0.5472 & -21.0894 & -11.1235 & 1.1300 & 1.6776 \\ -0.5148 & -0.2089 & -6.9263 & 0.6320 & -1.1522 & 0.5116 \end{bmatrix}$$

The closed loop system responses with the above controllers with the initial conditions considered in Section 3.6.1 are given in Figure 6.3 . The improvement in system performance is clear from these figures.

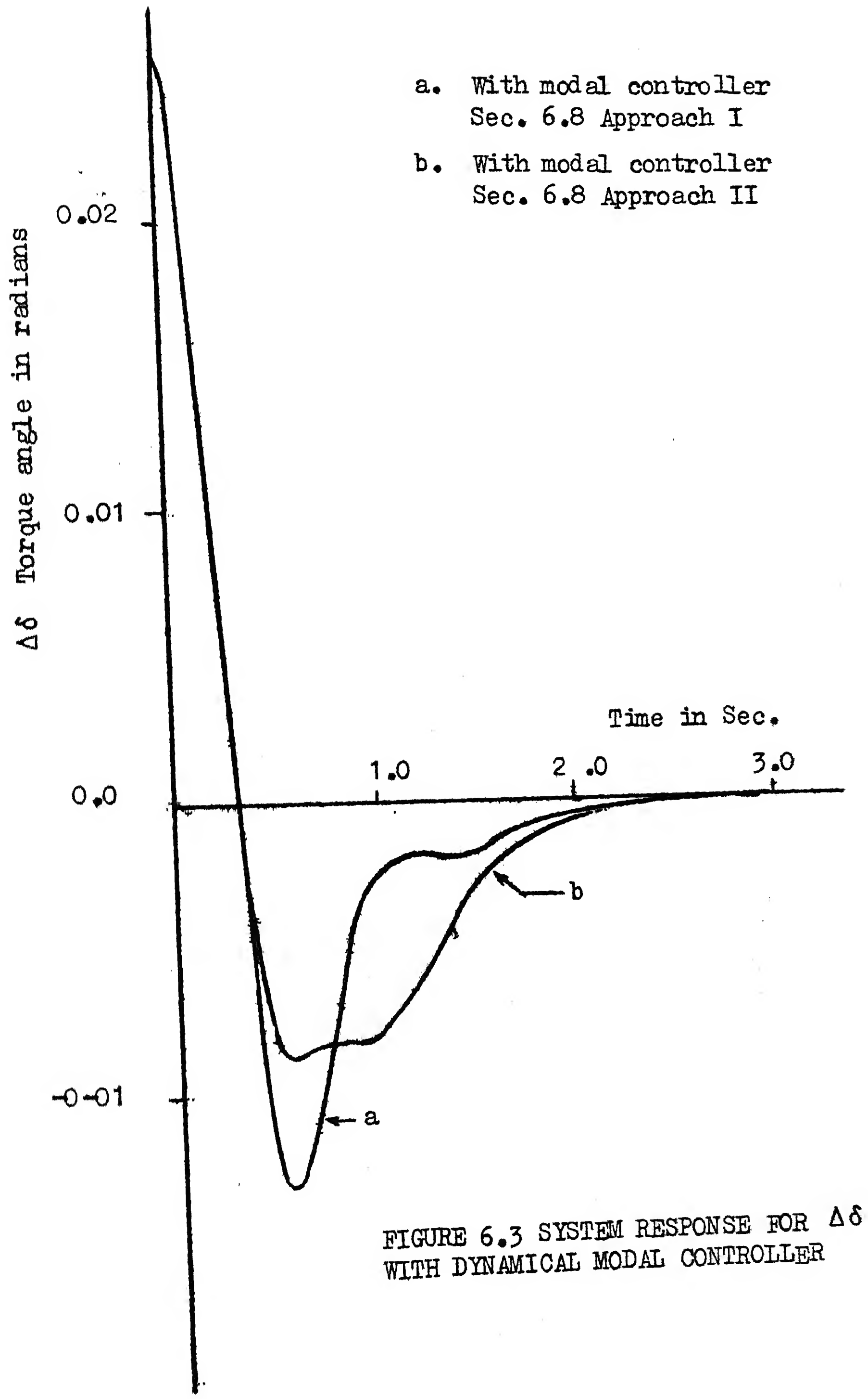

FIGURE 6.3 SYSTEM RESPONSE FOR Δδ WITH DYNAMICAL MODAL CONTROLLER

6.9 CONCLUSION

Techniques for designing accessible state feedback and output feedback modal controllers for linear, time invariant, multi-input-multi-output systems have been developed. Procedures for optimization of controller gains have been indicated. The assumptions made in the development of the algorithm do not seem to be overly restrictive. A method for designing dynamic modal controllers, based on the work of Liou et al [89], has also been developed. This is computationally more attractive than the procedures given in reference [89], especially for high order systems. Numerical examples illustrating the application of above procedures have been given.

CHAPTER 7

CONCLUSIONS

7.1 GENERAL

The basic aim of this thesis has been to develop suitable techniques of designing controllers which improve the dynamical behaviour of power systems. These techniques should be such that the designer has a 'feel' for the problem and his engineering judgement can be better utilized. The work reported and the contribution made in this thesis are reviewed in this chapter. Suggestions and scope for future work in this area have also been indicated.

7.2 REVIEW OF THE WORK DONE

Following a brief survey of the techniques available in the literature for designing control schemes for improving the dynamic performance of power systems, the state space descriptions of the power systems under investigation are discussed in Chapter 2. In Chapter 3, a procedure for designing regulators based on modal control theory with an optimality criterion has been suggested to obtain improved performance of linear time invariant dynamical systems. The algorithm developed is completely general, independent of the system structure and computationally easy. However, the dyadic structure assumed for modal controller matrix may be restrictive and may result in

large feedback gains especially in high order systems with large number of inputs. An algorithm based on the grouping of the dominant eigenvalues is therefore proposed in Chapter 4 resulting in lower feedback gains even for high order systems. A criterion for dividing eigenvalues into groups has been given. Procedures for the design of sub-optimal and near-optimal modal controllers requiring smaller computational efforts have also been developed. A technique is suggested to identify dominant control inputs for assigning the eigenvalues in a particular group. The feedback gains are marginally affected when feedback is arranged to dominant inputs. The techniques developed in the chapter are demonstrated for a large dynamical system described by fortyone differential equations. An effective method based on eigenvalue sensitivity to design modal controllers for improving the system performance over a wide range of operating conditions is developed in Chapter 5. This wide range modal controller reduces the variations in the closed loop system eigenvalue locations with the change in operating condition. Techniques for designing accessible state feedback and output feedback modal controllers for linear time invariant multi-input multi-output systems have been presented in Chapter 6. Procedure for optimization of controller feedback gains is suggested. A method for designing dynamic modal controller is also reported.

7.3 SUGGESTION AND SCOPE FOR FUTURE WORK

7.3.1 Assignment of Zeros in Addition to Poles

In this thesis the improvement in the power system performance is obtained by assigning the dominant eigenvalues to favourable locations. However, there is no control on the location of zeros of the closed loop system which also affect the system dynamic performance. Therefore it is desirable to develop techniques to assign both the poles and zeros to favourable locations.

7.3.2 Multimachine Systems

Examples of single machine connected to infinite bus are considered to illustrate the techniques developed in this thesis. It will be worthwhile to consider a realistic multi-machine power system to investigate the suitability of modal control techniques.

7.3.3 Attainable Pole Placement with Output Feedback

The algorithm developed to design output feedback modal controller for linear systems requires certain conditions to be satisfied. It may not always be possible to meet these requirements. Therefore it is desirable to investigate the attainable pole placement positions with a given output matrix.

7.3.4 Decentralized Control

The modal control law requires the feedback of

all the variables. In a modern power system which consists of large number of interconnected subsystems (Areas) it may be difficult and uneconomical to arrange feedback from the other areas to a controller in a particular area. It is worthwhile to investigate whether it is possible to improve the system performance by local control, that is, information available in a particular area is fed to its control inputs.

7.3.5 Non-linear Model

The performance equations are linearized around an operating point and control scheme is designed to improve the performance of this system for small disturbance. It is desirable to investigate the performance of non-linear model with the linear regulator for large disturbances.

7.3.6 Stochastic Control

In the studies reported in this thesis, a disturbance of deterministic type is assumed. In practice the type of disturbances are usually stochastic in nature. The problem of stochastic control with state dependent noise and unknown noise statistics is worth studying.

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APPENDIX A

MODAL CONTROL

Consider the system

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{A.1}$$

and its related single input system

$$\dot{\underline{x}} = A\,\underline{x} + \underline{\sigma}\,z \tag{A.2}$$

where vector $\underline{\sigma}$ is generated from the columns of the control matrix B such that

$$\underline{\sigma} = \sum_{j=1}^{m} \alpha_j\,\underline{b}_j = B\,\underline{\alpha} \tag{A.3}$$

where α_j's are scalars and $\underline{b}_j$'s are the columns of the control matrix B.

Consider the transformation

$$\underline{x} \triangleq W\,\underline{y} \tag{A.4}$$

where $W = [\underline{w}_1, \underline{w}_2, \ldots, \underline{w}_n]$, the eigenvector matrix A.
Let $V = [\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_n]$, be the reciprocal eigenvector matrix of A and the eigenvectors are normalized such that

$$V^T = W^{-1} \tag{A.5}$$

Then

$$\begin{aligned} \Lambda &\triangleq V^T A\,W \\ &= \mathrm{Dia}[\lambda_1, \lambda_2, \ldots, \lambda_n] \end{aligned} \tag{A.6}$$

Using the transformation (A.4), the system (A.2) is transformed as

$$\dot{\underline{y}} = \Lambda \underline{y} + V^T \underline{\sigma}\, z$$

$$= \Lambda \underline{y} + \underline{p}\, z \qquad (A.7)$$

where $\underline{p} = V^T \underline{\sigma}$

Consider a feedback control law of the form

$$z = \sum_{i=1}^{\ell} k_i\, y_i = \sum_{i=1}^{\ell} k_i < \underline{v}_i, \underline{x} >$$

$$= \underline{g}^T \underline{x} \qquad (A.8)$$

where the gain vector $\underline{g} = \sum_{i=1}^{\ell} k_i\, \underline{v}_i$ and k_i's are the controller gains and $\underline{v}_i$'s are used as the measurement vectors.

The closed loop system is

$$\dot{\underline{y}} = \bar{\Lambda}\, \underline{y}$$

where $\bar{\Lambda} = \Lambda + \underline{p}\, \underline{k}^T$

$\underline{k} = [k_1, k_2, \ldots, k_\ell, 0, \ldots, 0]^T$ is an n-vector

and k_i is the i^{th} controller gain and

$$\bar{\Lambda} = \begin{bmatrix} \bar{\Lambda}_{11} & 0 \\ \bar{\Lambda}_{21} & \bar{\Lambda}_{22} \end{bmatrix}$$

$$\bar{\Lambda} = \left[\begin{array}{cccccc|cc} \lambda_1+d_{11} & d_{12} & \cdots & & & & & \\ d_{21} & \lambda_2+d_{22} & \cdots & & & & 0 & \\ \cdots & \cdots & \cdots & \cdots & & & & \\ \cdots & \cdots & \cdots & \cdots & & & & \\ d_{\ell 1} & d_{\ell 2} & \cdots & \lambda_\ell+d_{\ell\ell} & & & & \\ \hline d_{\ell+1,1} & d_{\ell+1,2} & \cdots & d_{\ell+1,\ell} & & \lambda_{\ell+1} & & 0 \\ \cdots & \cdots & \cdots & \cdots & & & & \\ \cdots & \cdots & \cdots & \cdots & & & & \\ d_{n1} & d_{n2} & \cdots & d_{n\ell} & & 0 & & \lambda_n \end{array}\right] \tag{A.9}$$

where $d_{ij} = p_i\, k_j$

if $d_{ij} = 0$, then the i^{th} mode is uncontrollable.

The characteristic equation $\mathrm{Det}| sI - \bar{\Lambda}| = 0$ decomposes into

$$\mathrm{Det}\,| sI - \bar{\Lambda}_{11} | \prod_{i=\ell+1}^{n} (s - \lambda_i) = 0 \tag{A.10}$$

Thus the first 'ℓ' eigenvalues are variant and the rest $(n-\ell)$ eigenvalues remain invariant. If the eigenvalues of the controlled system are $(\rho_1, \rho_2, \ldots, \rho_\ell, \lambda_{\ell+1}, \ldots, \lambda_n)$, then

$$\mathrm{Det}\,| sI - \bar{\Lambda}_{11} | = (s-\rho_1)(s-\rho_2) \ldots (s-\rho_\ell) = 0$$

or $$\mathrm{Det}\,| \lambda_j I - \bar{\Lambda}_{11} | = 0, \; (\lambda_j = \rho_j, \; j = 1,2,\ldots,\ell) \tag{A.11}$$

The determinantal equations after simplification yield

$$k_j = \Big[\prod_{i=1}^{\ell} (\rho_i - \lambda_j) \Big] \Big/ \Big[p_j \prod_{\substack{i=1 \\ i\neq j}}^{\ell} (\lambda_i - \lambda_j) \Big], \; (j=1,2,\ldots,\ell) \tag{A.12}$$

Substituting $p_j = \underline{v}_j^T \underline{\sigma}$ and $\underline{\sigma} = B\,\underline{\alpha}$, the eqn.(A.12) reduces to

$$k_j = \Big[\prod_{i=1}^{\ell} (\rho_i - \lambda_j)\Big] \Big/ \Big[\underline{v}_j^T B\,\underline{\alpha} \prod_{\substack{i=1 \\ i \neq j}}^{\ell} (\lambda_i - \lambda_j)\Big] \qquad \text{(A.13)}$$

and

$$k_{j+1} = k_j^* \quad \text{if} \quad \lambda_{j+1} = \lambda_j^*$$

The gain vector $\underline{g}$ and the control law z are realized from eqn.(A.8). A real control law is obtained for shifting the complex eigenvalues in conjugate pairs. The control law for multi-input case is then realized as

$$\underline{u} = \underline{\alpha}\, z \qquad \text{(A.14)}$$

and

$$\underline{u} \triangleq F\,\underline{x}$$

Then the modal controller matrix

$$F = \underline{\alpha}\, \underline{g}^T \,. \qquad \text{(A.15)}$$

APPENDIX B

LAGRANGE MULTIPLIER TECHNIQUE

Case 1: Let the function J to be minimized is J_2 (eqn.3.10).

$$J = \sum_{j=1}^{m} \alpha_j^2 + \sum_{i=1}^{n} g_i^2 \qquad (B.1)$$

and the equality constraints to be satisfied are

$$\underline{g} - \sum_{i=1}^{\ell} k_i \underline{v}_i = \underline{0} \qquad (B.2)$$

where $\underline{v}_i$ and k_i are defined in (A.13).

Using Lagrange multiplier technique for the optimization problem, the Lagrangian function $\mathcal{L}_f$ is formed by augmenting the function J with the equality constraints multiplier by the respective Lagrangian multiplier μ_i, that is

$$\mathcal{L}_f = \sum_{j=1}^{m} \alpha_j^2 + \sum_{i=1}^{n} g_i^2 + \underline{\mu}^T[\underline{g} - \sum_{j=1}^{\ell} k_j \underline{v}_j] \qquad (B.3)$$

where the elements μ_i, $(i = 1,2,\ldots,n)$ of the $\underline{\mu}$ are the 'n' Lagrange multipliers. It follows from eqn.(B.3) that the optimum value of $\mathcal{L}_f$, when the equality constraints (eqn. B.2) are satisfied, is equal to the optimum value of J. Then it follows

$$\frac{\partial \mathcal{L}_f}{\partial \mu_i} = [g_i - \sum_{j=1}^{\ell} k_j v_j^{(i)}] = 0 \quad , \quad (i = 1,2,\ldots,n) \qquad (B.4)$$

$$\frac{\partial \mathcal{L}_f}{\partial g_i} = 2g_i + \mu_i = 0 \quad , \quad (i = 1,2,\ldots,n) \qquad (B.5)$$

$$\frac{\partial \mathcal{L}_f}{\partial \alpha_i} = 2\alpha_i + \underline{\mu}^T \left[\sum_{j=1}^{\ell} \left[\left(\prod_{p=1}^{\ell} (\rho_p - \lambda_j) \right) \underline{v}_j^T \underline{b}_i \underline{v}_j \right] \Big/ \left[\left(\prod_{\substack{p=1 \\ p \neq j}}^{\ell} (\lambda_p - \lambda_j) \right) (\underline{v}_j^T B \underline{\alpha})^2 \right] \right.,$$

$$(i = 1,2,\ldots,m) \qquad \text{(B.6)}$$

where

$v_j^{(i)}$ is the i^{th} element of $\underline{v}_j$ and

$\underline{b}_i$ is the i^{th} column of the control matrix B.

At optimum value of J and $\mathcal{L}_f$ eqns. (B.4) to (B.6) must be equal to zero. The negative gradient of the Lagrangian function $\mathcal{L}_f$ with respect to $\underline{\alpha}$ is

$$(\Delta \underline{\alpha} \triangleq - (\partial \mathcal{L}_f / \partial \underline{\alpha}) \qquad \text{(B.7)}$$

Case 2: Let the function J to be minimized in J_1(eqn.3.8).

$$J = \sum_{j=1}^{m} \sum_{i=1}^{n} \alpha_j^2 \, g_i^2 \qquad \text{(B.8)}$$

Then the Lagrangian function is

$$\mathcal{L}_f = \sum_{j=1}^{m} \sum_{i=1}^{n} \alpha_j^2 \, g_i^2 + \underline{\mu}^T [\underline{g} - \sum_{j=1}^{\ell} k_j \underline{v}_j] \qquad \text{(B.9)}$$

Then it follows

$$\frac{\partial \mathcal{L}_f}{\partial \mu_i} = [g_i - \sum_{j=1}^{\ell} k_j \, v_j^{(i)}], \quad (i = 1,2,\ldots,n) \qquad \text{(B.10)}$$

$$\frac{\partial \mathcal{L}_f}{\partial g_i} = \sum_{j=1}^{m} 2\alpha_j^2 \, g_i + \mu_i \quad , \quad (i=1,2,\ldots,n) \tag{B.11}$$

$$\frac{\partial \mathcal{L}_f}{\partial \alpha_i} = \sum_{j=1}^{n} 2\alpha_i \, g_j^2 + \mu^T \left[\sum_{j=1}^{\ell} \left[\left(\prod_{p=1}^{\ell} (\rho_p - \lambda_j) \right) p_{ji} \, \underline{v}_j \right] \Big/ \left[\left(\prod_{\substack{p=1 \\ p \neq j}}^{\ell} (\lambda_p - \lambda_j) \right) (\underline{v}_j^T \, B \, \underline{\alpha})^2 \right] \right]$$

$$(i = 1,2,\ldots,m) \tag{B.12}$$

The negative gradient of the Lagrangian function $\mathcal{L}_f$ with respect to $\underline{\alpha}$ is

$$(\Delta \underline{\alpha}) \stackrel{\Delta}{=} -(\partial \mathcal{L}_f / \partial \underline{\alpha}) \, . \tag{B.13}$$

APPENDIX C

LARGE CHEMICAL PLANT

A generalised chemical plant model of thirty first order was postulated by Williams and Otto [78] for the investigation of computer control. Davison and Chadha [73] modified the above model by increasing its order to forty one by considering four plates of the distillation columns instead of two plates considered earlier to give a realistic model. The description of the plant as well as that of linearized mathematical model with data is given in detail in reference [73].

BRIEF DESCRIPTION OF PLANT

The basic units of the chemical plant are a reactor, a heat exchanger, a decanter and a distillation unit complete with reboiler and condenser. Two pure feeds and one recycle feed are provided to the reactor. Six materials are present in the effluent from the reactor. The exothermic reactions are controlled by the addition or substraction of heat to the reactor.

Each of the reactor and heat exchanger units are represented by a set of six differential equations in the mathematical model. Five differential equations are required to represent decanter unit and the distillation plant is

represented by a set of twenty differential equations. The interconnections are modelled by four differential equations. In all forty one differential equations represent the chemical plant.

DATA FOR THE LINEAR MODEL OF THE PLANT

The data for the linear model

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u}$$

as given in reference [73] are as follows:

The system matrix A is given by:

$$A = (a_{i,j}) \quad , \quad i=1,2,\ \ldots,\ 41$$
$$j=1,2,\ \ldots,\ 41$$

$a_{1,1} = -40.74$, $a_{1,2} = -6.36$, $a_{2,1} = -20.1$, $a_{2,2} = -31.47$,

$a_{2,3} = -68.4$, $a_{3,1} = 40.2$, $a_{3,2} = 3.78$, $a_{3,3} = -182.44$,

$a_{3,5} = -7.35$, $a_{4,2} = 8.94$, $a_{4,3} = 136.8$, $a_{4,4} = -20.64$,

$a_{5,2} = 4.47$, $a_{5,3} = 55.9$, $a_{5,5} = -24.315$, $a_{6,3} = 37.5$,

$a_{6,5} = 11.025$, $a_{6,6} = -20.64$, $a_{7,1} = 3030.0$, $a_{7,2} = 1449.5$,

$a_{7,3} = 8168.0$, $a_{7,5} = 190.6$, $a_{7,7} = -35.82$, $a_{8,7} = 436.0$,

$a_{8,8} = -603.4$, $a_{8,9} = 167.0$, $a_{9,8} = 17.7$, $a_{9,9} = -202.69$,

$a_{21,8} = 16.42$, $a_{21,21} = -16.42$, $a_{22,22} = -1.56$,

$a_{23,23} = -1.04$, $a_{24,24} = -2.08$, $a_{25,25} = -3.64$, $a_{26,26} = -3.64$,

$a_{22,27} = 1.56$, $a_{23,28} = 1.04$, $a_{24,29} = 2.08$, $a_{25,30} = 3.64$,

$a_{26,31} = 3.64$

and the following algorithms give the rest of the non-zero elements.

$$a_{i-36,i} = 10.75 \quad , \quad i=37,38,..,41$$

$$a_{i,i} = -350.0 \quad , \quad i=10,11,..,15$$

$$a_{i+9,i} = 350.0 \quad , \quad i=1,2,..,6$$

$$a_{i+6,i} = 13.7 \quad , \quad i=10,11,..,14$$

$$\left.\begin{array}{l} a_{i,i} = -13.2 \\ a_{i+11,i} = 15.6 \end{array}\right] \quad , \quad i=16,17,..,20$$

$$\left.\begin{array}{l} a_{i+5,i+5} = -46.0 \\ a_{i+10,i+10} = -53.95 \\ a_{i+15,i+15} = -35.65 \\ a_{i+5,i} = 14.8 \\ a_{i+5,i+10} = 15.6 \\ a_{i+10,i+5} = 35.65 \end{array}\right] \quad , \quad i=22,23,..,26$$

$$\left.\begin{array}{l} a_{i+10,i+15} = 18.3 \\ a_{i+15,i+15} = 35.65 \end{array}\right] \quad , \quad i=22,23,..,26$$

The control matrix B is given by

$$B = (b_{i,j}) \; , \quad \begin{array}{l} i = 1,2,..,\ 41 \\ j = 1,2,..,\ 8 \end{array}$$

and its non-zero elements are:

$b_{1,1} = 2.16\times10^{-4}$, $b_{2,2} = b_{1,1}$, $b_{1,8} = 2.87 \times 10^{-5}$,

$b_{7,1} = 1.15\times10^{-2}$, $b_{7,2} = b_{7,1}$, $b_{7,3} = 1.5$, $b_{7,4} = 1.5$,

$b_{7,5} = 3.36$, $b_{9,6} = 185.0$, $b_{9,7} = -2.4\times10^{-2}$, $b_{2,8} = 9.07\times10^{-5}$,

$b_{3,8} = 0.582\times10^{-5}$, $b_{4,8} = 8.21 \times 10^{-5}$, $b_{5,8} = 0.819 \times 10^{-5}$,

$b_{7,8} = -1345.0 \times 10^{-5}$.

APPENDIX D

EIGENVALUE SENSITIVITY [80]

Let $f(s,A)$ be the characteristic polynomial of A. Then

$$f\,(s,A_o) = \det\,[I_s - A_o].$$

Consider the variation dA in the nominal value A_o and let the eigenvalue differential be ds, due to differential change dA.

On expanding $f(s,A) = 0$ in Tayler series about nominal value A_o, we obtain

$$f(s,A) = [f(s,A)]_o + \sum_{k=1}^{\infty} \frac{1}{k!}\,[d^k\,f(s,A)]_o = 0 \qquad (D.1)$$

where $d^k(.)$ is the k^{th} differential of (.).

Since $[f(s,A)]_o = f(s,A_o) = 0$, the eqn. (D.1) reduces to

$$f(s,A) = \sum_{k=1}^{\infty} \frac{1}{k!}\,[d^k f(s,A)]_o = 0 \qquad (D.2)$$

From matrix theory [90], it can be shown that for any square matrix P

$$d\,[\det P] = \operatorname{adj} P \circledast dP \qquad (D.3)$$

and the inner product of two equidimensional square matrices is defined as

$$C \circledast D = \sum_i c_i\,d_i \;=\; \mathrm{tr}\,[C\,D]$$

where c_i is the i^{th} row of C and d_i is the i^{th} column of D.

Then from eqn. (D.2)

$$\sum_{k=1}^{\infty} \frac{1}{k!}\,[d^{k-1}\,[\operatorname{adj}\,(sI-A) \circledast d\,(Is-A)]]_o = 0 \qquad (D.4)$$

If the differential change is linear, then $d^kA = 0$ for $k > 1$ and if the variation in parameters are sufficiently small then the terms

$$\sum_{k=2}^{\infty} \frac{1}{k!} [d^{k-1}[adj(sI-A) \circledast d(Is-A)]]_{so}$$

are small being the product of differentials. Then the eqn. (D.4) is rewritten as

$$[adj\ (sI-A)]_o \circledast Ids = [adj\ (sI-A)]_o \circledast dA \tag{D.5}$$

Since adj $(sI-A) \circledast Ids = Tr.[adj\ (sI-A)]\ ds$, where $Tr.(.)$ is the trace of the matrix (.), the eqn. (D.5) reduces to

$$Tr.[adj\ (sI-A)]_o\ ds = [adj\ (sI-A)]_o \circledast dA$$

and for a particular s_i the above equation yields

$$ds_i = [Tr.\ [adj\ (sI-A)]_o]^{-1}\ [[adj\ (sI-A)]_o \circledast dA]$$

$$= [Tr.\ R^c(\lambda_i^d)]_o^{-1}\ [R^c\ (\lambda_i^d)]_o \circledast [d\ (A + \underline{b}\ \underline{f}_o^T)]$$

or
$$\frac{ds_i}{d\theta_j} = [Tr.\ R^c(\lambda_i^d)]_o^{-1}\ [R^c(\lambda_i^d)]_o \circledast [\frac{d}{d\theta_j}\ (A + \underline{b}\underline{f}_o^T)],$$

$$(i=1,2,\ldots,n),\ (j=1,2,\ldots,r) \tag{D.6}$$

where $R^c(\lambda_i^d) \triangleq adj\ [\lambda_i^d I - A]$ and

λ_i^d is the i^{th} desired closed loop system eigenvalue.

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